1\. (a) $\alpha + \beta = p$ [illegible] $\alpha\beta = q$

$\alpha(\alpha + \beta)$ [illegible] $\beta(\alpha + \beta)$ [illegible]

[illegible] $x^2 - ax + b = 0$ [illegible]

[illegible] $a = \alpha(\alpha + \beta) + \beta(\alpha + \beta)$

$= (\alpha + \beta)^2 = p^2$ [illegible]

$b = [\alpha(\alpha + \beta)][\beta(\alpha + \beta)]$

$= \alpha\beta(\alpha + \beta)^2 = qp^2$

$\therefore$ [illegible]

$x^2 - p^2x + qp^2 = 0$

(b) $\therefore$ [illegible] x, y [illegible]

$2x^2 + \lambda xy + 3y^2 - 5y - 2 \equiv$

$(ax + by + c)(px + qy + r)$ [illegible]

$x = 0$ [illegible] $3y^2 - 5y - 2$

$\equiv (by + c)(qy + r)$ [illegible] y [illegible]

[illegible] $3y^2 - 5y - 2 = (3y + 1)(y - 2)$

$\therefore b = 3,\ c = 1,\ q = 1$ [illegible] $r = -2$

$\therefore$ [illegible] x, y [illegible]

$2x^2 + \lambda xy + 3y^2 - 5y - 2 \equiv$

$(ax + 3y + 1)(px + y - 2)$ [illegible]

[illegible] $y = 0$ [illegible] x [illegible]

$2x^2 - 2 \equiv (ax + 1)(px - 2)$

[illegible]

$2 = ap$ [illegible] $-2a + p = 0$

$a^2 = 1 \Rightarrow a = \pm 1$ [illegible]

$p = \pm 2$ [illegible]

$\therefore 2x^2 + \lambda xy + 3y^2 - 5y - 2$

$= (-x + 3y + 1)(-2x + y - 2)$ [illegible]

$= (x + 3y + 1)(2x + y - 2)$ [illegible]

xy [illegible] $\lambda = \pm 7$ [illegible]

(c) $\dfrac{2x^3 - x + 3}{x(x-1)^2} = A + \dfrac{B}{x} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2}$

[illegible] A, B, C [illegible] D [illegible]

$$\frac{2x^3 - x + 3}{x(x-1)^2} = \frac{A(x^3 - 2x^2 + x) + B(x^2 - 2x + 1) + C(x^2 - x) + Dx}{x(x-1)^2}$$

[illegible]

$A = 2,\ -2A + B + C = 0,$
$A - 2B - C + D = -1 \Rightarrow B = 3$

[illegible] $C = 1$ [illegible] $D = 4$ [illegible]

$\therefore \dfrac{2x^3 - x + 3}{x(x-1)^2} = 2 + \dfrac{3}{x} + \dfrac{1}{x-1} + \dfrac{4}{(x-1)^2}$

2\. (a) $U_n = 1.n + 2(n-1) + \ldots\ldots +$
$(n-1).2 + n.1$ [illegible]

$U_n = \dfrac{1}{6}n(n+1)(n+2)$ —— ❶

[illegible]

$n = 1$ [illegible]

❶ [illegible] L. H. S $= U_1 = 1.1 = 1$

❶ [illegible] R. H. S. $= \dfrac{1}{6}.1.2.3 = 1$

$\therefore$ ❶ [illegible] L.H.S = ❶ [illegible] R.H.S.

$\therefore n = 1$ [illegible]

$n = p$ [illegible]

[illegible] $U_p = \dfrac{1}{6}p(p+1)(p+2)$

$\therefore n = p + 1$ [illegible] U_{p+1}

$= 1.(p+1) + 2.p + \ldots + p.2 + (p+1).1$

$= 1.p + 2(p-1) + \ldots\ldots + p.1$
$+ (1 + 2 + \ldots\ldots + (p+1)).1$

$= U_p + (1 + 2 + \ldots\ldots + (p+1))$

$= \dfrac{1}{6}p(p+1)(p+2) + \dfrac{1}{2}(p+1)(p+2)$

$= \dfrac{1}{6}(p+1)(p+2)(p+3)$

$\therefore n = p + 1$ [illegible]
[illegible]
[illegible] n [illegible]

$U_n = \dfrac{1}{6}n(n+1)(n+2)$ [illegible]

[illegible] n [illegible]

$\dfrac{1}{U_n} = \dfrac{6}{n(n+1)(n+2)}$

$= \dfrac{3}{n(n+1)} - \dfrac{3}{(n+1)(n+2)}$

$= V_n - V_{n+1}$

[illegible] $V_n = \dfrac{3}{n(n+1)}$

$\dfrac{1}{U_1} = V_1 - V_2$

$\dfrac{1}{U_2} = V_2 - V_3$

....
....
....

$\dfrac{1}{U_{n-1}} = V_{n-1} - V_n$

$\dfrac{1}{U_n} = V_n - V_{n+1}$

$\sum_{r=1}^{n} \frac{1}{U_r} = V_1 - V_{n+1}$

$= \frac{3}{2} - \frac{3}{(n+1)(n+2)}$

$\sum_{r=1}^{\infty} \frac{1}{U_r} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{U_r}$

$= \lim_{n\to\infty} \left(\frac{3}{2} - \frac{3}{(n+1)(n+2)} \right)$

$= \frac{3}{2}$ //

$\because \lim_{n\to\infty} \frac{3}{(n+1)(n+2)} = 0$

(ii) $(1 + kx)^{10}$

$= {}^{10}C_0 + {}^{10}C_1 kx + {}^{10}C_2 k^2x^2 + \ldots\ldots$
$+ {}^{10}C_r k^r x^r + \ldots + C_{10} k^{10} x^{10}$

$= a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{10} x^{10}$

$\therefore \sum_{r=0}^{10} {}^{10}C_r k^r x^r = \sum_{r=0}^{10} a_r x^r, \quad \forall x \in R$

$\therefore {}^{10}C_r k^r = a_r,\ r = 0, 1, 2, \ldots\ldots 10$ [illegible]

${}^{10}C_2 k^2 = a_2 = \frac{20}{9}$ [illegible]

[illegible] ${}^{10}C_2 k^2 = \frac{10.9}{2} k^2$

$\Rightarrow \frac{10.9}{2} k^2 = \frac{20}{9}$

$\therefore k^2 = \left(\frac{2}{9}\right)^2 \Rightarrow k = \frac{2}{9}$

($\because$ k [illegible])

$\therefore \left(1 + \frac{2}{9}x\right)^{10}$

$= a_0 + a_1 x + a_2 x^2 + \ldots a_{10} x^{10}$

$x = 1$ [illegible]

$\left(\frac{11}{9}\right)^{10} = a_0 + a_1 + a_2 + \ldots + a_{10}$ —— ❶

$x = -1$ [illegible]

$\left(\frac{7}{9}\right)^{10} = a_0 - a_1 + a_2 - \ldots - a_9 + a_{10}$ —— ❷

❶ - ❷ $\left(\frac{11}{9}\right)^{10} - \left(\frac{7}{9}\right)^{10}$

$= 2\left[a_1 + a_3 + \ldots\ldots + a_9\right]$

[illegible]

$a_1 + a_3 + a_5 + a_7 + a_9 = \frac{11^{10} - 7^{10}}{2\,9^{10}}$ —— ❸

❷ + ❸

$\Rightarrow a_0 + a_2 + a_4 + a_6 + a_8 + a_{10} = \left(\frac{7}{9}\right)^{10} + \frac{11^{10} - 7^{10}}{2 \times 9^{10}}$

$= \frac{2 \times 7^{10} + 11^{10} - 7^{10}}{2 \times 9^{10}}$

$= \frac{11^{10} + 7^{10}}{2 \times 9^{10}}$ //

03. (i) $\frac{(-1+i)^3}{(1+i)^4} = \frac{(-1+i)^3(1-i)^4}{(1+i)^4(1-i)^4}$

$= -\frac{(1-i)^7}{2^4}$

$= -\frac{1}{2^4}(1 - 7i - 21$

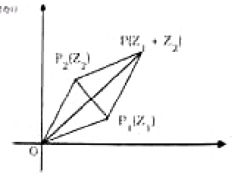
$+ 35i + 35 - 21i - 7 + i)$

$= -\frac{1}{2} - \frac{1}{2}i$

$= \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

$\therefore$ [illegible] $\frac{1}{\sqrt{2}}$ [illegible] $\frac{5\pi}{4}$

(ii)



P_1, P_2 [illegible] Z_1 [illegible] Z_2 [illegible] OP_1 [illegible] OP_2 [illegible] OP_1PP_2 [illegible] P [illegible] $Z_1 + Z_2$ [illegible]

[illegible] -

$Z_1 = x_1 + iy_1$ [illegible] $Z_2 = x_2 + iy_2$

[illegible]

[illegible] $P_1 = (x_1, y_1)$ [illegible] $P_2 = (x_2, y_2)$ [illegible]

$P = (x, y)$ [illegible]

[illegible] $\frac{x+0}{2} = \frac{x_1 + x_2}{2}$ [illegible]

$\frac{y+0}{2} = \frac{y_1 + y_2}{2}$

$\Rightarrow x = x_1 + x_2$ [illegible] $y = y_1 + y_2$

$\therefore P = (x_1 + x_2, y_1 + y_2)$

P [illegible] $(x_1 + x_2) + i(y_1 + y_2)$ [illegible]

[illegible] P [illegible] $(x_1 + iy_1) + (x_2 + iy_2)$ [illegible]

[illegible] P [illegible] $Z_1 + Z_2$ [illegible]

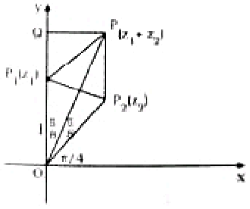
$Z_1 = \frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)}$

$= \frac{1 + 2i + i^2}{1 - i^2} = \frac{2i}{2} = i$

$Z_2 = \frac{\sqrt{2}}{1-i} = \frac{1}{\sqrt{2}}(1+i)$

$= 1(\cos \pi/4 + i \sin \pi/4)$

$Z_1 + Z_2 = \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}} + 1\right) i$



$\tan \frac{\pi}{8} = \frac{QP}{OQ}$

$= \frac{Re(z_1 + z_2)}{Im(z_1 + z_2)} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + 1}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{1 + \sqrt{2}}{\sqrt{2}}} = \frac{1}{1 + \sqrt{2}}$

$= \frac{1 - \sqrt{2}}{1 - 2} = \sqrt{2} - 1$ //

04. (i) $\lim_{x\to 0} \frac{1 - \cos^2(2 \sin x)}{2 \sin^2 x}$

$= \lim_{x\to 0} \frac{2 \sin^2(2 \sin x)}{(2 \sin x)^2}$

$= \lim_{x\to 0} 2\left[\frac{\sin(2 \sin x)}{2 \sin x}\right]^2$

$= 2 \lim_{t\to 0} \left(\frac{\sin t}{t}\right)^2$

$= 2$ // [illegible] $t = 2\sin x$; $x \to 0$ [illegible] $t \to 0$ [illegible]

(ii) $y = e^{k\sin^{-1}x}$

$\frac{dy}{dx} = e^{k\sin^{-1}x} \times k \times \frac{1}{\sqrt{1 - x^2}}$

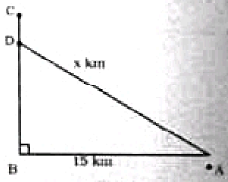
$\sqrt{1 - x^2}\ \frac{dy}{dx} = k\,e^{k\sin^{-1}x}$

$= ky$ //

$x = \frac{1}{2}$ [illegible] $y = e^{k\pi/6}$

[illegible] $\frac{dy}{dx} = \frac{2}{\sqrt{3}}\ k\,e^{k\pi/6}$ //

(iii)



$DC = 50 - \sqrt{x^2 - 225}\ ;\ x \geq 15$ [illegible]

$T(x) = \frac{x}{40} + \frac{50 - \sqrt{x^2 - 225}}{50}$

$\frac{dT(x)}{dx} = \frac{1}{40} - \frac{1}{2} \times \frac{2x}{50\sqrt{x^2 - 225}}$

$\frac{dT(x)}{dx} = \frac{1}{40} - \frac{x}{50\sqrt{x^2 - 225}}$

$= \frac{1}{200\sqrt{x^2 - 225}} \times \left\{5\sqrt{x^2 - 225} - 4x\right\}$

$= 0 \quad x = 25$ [illegible]

x හි අගය	15 < x < 25	x = 25	25 < x < 50
$\frac{dT(x)}{dx}$	(-)	0	(+)

x හි අගය 15 සිට 25 දක්වා වැඩිවන විට T(x) අඩුවේ.
x හි අගය 25 සිට 50 දක්වා වැඩිවන විට T(x) වැඩිවේ.
x හි අගය 25 විට T(x) අවම වේ.
∴ අවම කාලය T(25) වේ.

එනම් අවම කාලය $= \frac{25}{40} + 1 - \frac{\sqrt{625 - 225}}{50}$

$= \frac{5}{8} + 1 - \frac{20}{50}$

$= 0.625 + 1 - 0.400$

$= \underline{1.225}$ h

05. (අ) $I = \int_1^8 \frac{1}{x^{\frac{1}{3}} + x^{\frac{2}{3}}} dx$

$y = x^{\frac{1}{3}}$ ලෙස ගනිමු. එවිට $dy = \frac{1}{3} \frac{1}{x^{\frac{2}{3}}} dx$

එනම් $3y^2 dy = dx$

x = 1 විට y = 1, x = 8 විට y = 2

එවිට $I = 3\int_1^2 \frac{dy}{1 + y^2} = 3\left[\tan^{-1} y\right]_1^2$

$= 3\left\{\tan^{-1} 2 - \tan^{-1} 1\right\}$

$= 3\left\{\tan^{-1} 2 - \pi/4\right\}$

(ආ) $I = \int_0^{\pi} e^{-2x} \cos x \, dx$

$= \int_0^{\pi} e^{-2x} \frac{d}{dx}(\sin x) dx$

$= \left[e^{-2x} \underset{0}{\underset{\|}{\sin x}}\right]_0^{\pi} + 2\int_0^{\pi} e^{-2x} \sin x \, dx$

$= \underline{2J}$ —— ①

$J = \int_0^{\pi} e^{-2x} \sin x \, dx$

$= \int_0^{\pi} e^{-2x} \frac{d}{dx}(-\cos x) dx$

$= \left[-e^{-2x} \cos x\right]_0^{\pi} - 2\int_0^{\pi} e^{-2x} \cos x \, dx$

$= e^{-2\pi} + 1 - 2I$ —— ②

① හා ② න් $I = \frac{1}{5}(e^{2\pi} + 1)$

$I = \frac{2}{5}(e^{2\pi} + 1)$

(ඇ) $\frac{x^2 - 5x}{(x - 1)(x + 1)^2}$

$= \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$

මෙහි A, B සහ C නියත වේ.

∴ A = -1 සහ C = -3

x = 0 විට 0 = 1 + B - 3 ⇒ B = 2

$\int \frac{x^2 - 5x}{(x - 1)(x + 1)^2} dx$

$= -\int \frac{dx}{(x - 1)} + 2\int \frac{dx}{x + 1} - 3\int \frac{dx}{(x + 1)^2}$

$= -\ln|x - 1| + 2\ln|x + 1| + \frac{3}{x + 1} + D$

D යනු අභිමත නියතයකි.

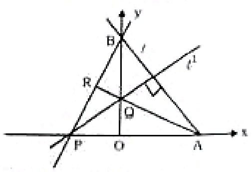
06. අවශ්‍ය රේඛාවේ සමීකරණය
y = mx + c ලෙස ගනිමු.

(a, 0) සහ (0, b) ලක්ෂ්‍ය හරහා යන නිසා
0 = am + c සහ b = c වේ.

$\therefore m = -\frac{b}{a}$

∴ රේඛාවේ සමීකරණය

$y = -\frac{b}{a}x + b \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$



l හි සමීකරණය $\frac{x}{h} + \frac{y}{k} = 1$ නිසා

A = (h, 0) සහ B = (0, k)

l^1 හි සමීකරණය

$\frac{x}{h^1} + \frac{y}{k^1} = 1$ ලෙස ගනිමු. $l \perp l^1$ නිසා

$\left(-\frac{k^1}{h^1}\right)\left(-\frac{k}{h}\right) = -1 \Rightarrow \frac{k^1}{h} = \frac{-h^1}{k} = t$ ලෙස ගනිමු

එවිට $k^1 = ht$, $h^1 = -kt$ සහ

එමගින් P = (-kt, 0) සහ Q = (0, ht)

ලෙස ලැබේ.

AQ හි සමීකරණය $\frac{y - 0}{x - h} = \frac{0 - ht}{h - 0} = -t$

$\Rightarrow y = -tx + ht$

$y + tx = ht$

BP හි සමීකරණය $\frac{y - k}{x - 0} = \frac{k - 0}{0 - (-kt)} = \frac{1}{t}$

$\Rightarrow yt - kt = x$

$\Rightarrow yt - x = kt$

$R \equiv (x_0, y_0)$ නම් එවිට $y_0 = t(h - x_0)$ සහ

$x_0 = t(y_0 - k)$

$\therefore \frac{y_0}{h - x_0} = \frac{x_0}{y_0 - k}$, $x_0 \neq h$ සහ $y_0 \neq k$ විට

එනම් $x_0^2 + y_0^2 - hx_0 - ky_0 = 0$

එමගින් R ලක්ෂ්‍යය $x^2 + y^2 - hx - ky = 0$ වෘත්තය මත වේ.

07. C_1 සහ C_2 වෘත්ත දෙකේ කේන්ද්‍ර ද r_1 සහ r_2 අනු වෘත්ත දෙකේ අරයන් ලෙස ගනිමු.

එවිට $C_1 \equiv (-g_1, -f_1)$; $r_1 = \sqrt{g_1^2 + f_1^2 - c_1}$ සහ

$C_2 \equiv (-g_2, -f_2)$; $r_2 = \sqrt{g_2^2 + f_2^2 - c_2}$

වෘත්ත දෙක ප්‍රලම්බව ඡේදනය වේ නම්

$(C_1C_2)^2 = r_1^2 + r_2^2$

එනම් $(g_1 - g_2)^2 + (f_1 - f_2)^2$

$= (g_1^2 + f_1^2 - c_1) + (g_2^2 + f_2^2 - c_2)$

එනම් $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

(p, 0) වෘත්තයේ කේන්ද්‍රය ද r වෘත්තයේ අරය ලෙස ගනිමු. එවිට වෘත්තයේ සමීකරණය

$(x - p)^2 + y^2 = r^2$

එනම් $x^2 + y^2 - 2px + p^2 - r^2 = 0$

$x^2 + y^2 - 8x - 6y + 21 = 0$ වෘත්තය ප්‍රලම්බව ඡේදනය කරන නිසා

$2(-p)(-4) = 21 + p^2 - r^2$

එනම් $r^2 = p^2 - 8p + 21$ —— ❶

තවද $x^2 + y^2 + 4x + 6y + 9 = 0$ වෘත්තය ස්පර්ශ කරන නිසා

$|\pm r + 2| = \sqrt{(p - (-2))^2 + (0 - (-3))^2}$

$= \sqrt{(p + 2)^2 + 3^2}$ —— ❷

$r^2 \pm 4r + 4 = p^2 + 4p + 4 + 9$

$r^2 \pm 4r = p^2 + 4p + 9$ —— ❸

❸ - ❶ ⇒

$r = \pm 3(p - 1)$ —— ❹

එවිට,

❶ න් $9p^2 - 18p + 9 = p^2 - 8p + 21$

$8p^2 - 10p - 12 = 0$

$4p^2 - 5p - 6 = 0$

$(4p + 3)(P - 2) = 0$

p = -¾ හෝ p = 2

P = 2 විට ❹ මත ලකුණ සැලකූවිට r = 3

P = -3/4 විට ❹ හි සෘණ ලකුණ සැලකූ විට

$r = \frac{21}{4}$

∴ P = 2, r = 3 විට ❷ න් ධන ලකුණ සැලකූවිට

$3 + 2 = \sqrt{(2 + 2)^2 + 9} = 5$

වෘත්ත දෙකේ අරයන්ගේ එකතුව

= කේන්ද්‍ර දෙක අතර දුර

∴ වෘත්ත දෙක බාහිරව ස්පර්ශ වේ.

එවිට වෘත්තයේ සමීකරණය

$(x - 2)^2 + y^2 = 9$

එනම් $x^2 + y^2 - 4x - 5 = 0$

$P = -\frac{3}{4}$ සහ $r = 21/4$ විට ❷ සෘණ සැලකූවිට

$\left|-\frac{21}{4} + 2\right| = \sqrt{\left(-\frac{3}{4} + 2\right)^2 + 9}$

$\frac{13}{4} = \sqrt{\frac{25}{16} + \frac{144}{16}}$

∴ වෘත්ත දෙකේ අරයන්ගේ අන්තරය

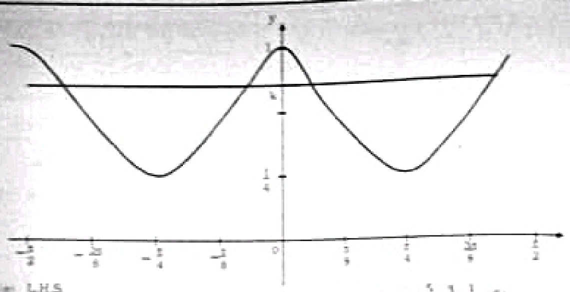
= කේන්ද්‍ර දෙක අතර දුර

∴ වෘත්ත දෙක අභ්‍යන්තරව ස්පර්ශ වේ.

∴ එවිට වෘත්තයේ සමීකරණය

$(x + ¾)^2 + y^2 = \left[\frac{21}{4}\right]^2$

එනම් $2x^2 + 2y^2 + 3x - 54 = 0$



(a) L.H.S

$= \frac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta}$

$= \frac{1 + \sin\theta + \cos\theta}{1 + \sin\theta - \cos\theta}$

$= \frac{(1 + \sin\theta + \cos\theta)^2}{(1 + \sin\theta)^2 - \cos^2\theta}$

$= \frac{(1+\sin\theta)^2 + 2\cos\theta(1+\sin\theta) + \cos^2\theta}{1 + 2\sin\theta + \sin^2\theta - \cos^2\theta}$

$= \frac{(1+\sin\theta)^2 + 2\cos\theta(1+\sin\theta) + (1+\sin\theta)(1-\sin\theta)}{2\sin\theta(1+\sin\theta)}$

$= \frac{1 + \sin\theta + 2\cos\theta + (1 - \sin\theta)}{2\sin\theta}$

$= \frac{2(1 + \cos\theta)}{2\sin\theta} = \frac{1 + \cos\theta}{\sin\theta}$

(b) $8(\cos^6 x + \sin^6 x)$

$= 8\{\sin^6 x + (1 - \sin^2 x)^3\}$

$= 8(\sin^6 x + 1 - 3\sin^2 x + 3\sin^4 x - \sin^6 x)$

$= 8\{1 - 3\sin^2 x + 3\sin^4 x\}$

$= 8 - 24\sin^2 x(1 - \sin^2 x)$

$= 8 - 6\sin^2 2x$

$= 8 - 3(1 - \cos 4x)$

$= 5 + 3\cos 4x$

$y = \cos^6 x + \sin^6 x$ [illegible]

$y = \frac{5}{8} + \frac{3}{8}\cos 4x$ [illegible]

y [illegible] $= \frac{8}{8} = 1$ [illegible]

$\cos 4x = 1 \Rightarrow x = 0, -\pi/2, \pi/2$

y [illegible] $= \frac{5}{8} - \frac{3}{8} = \frac{1}{4}$ [illegible]

$\cos 4x = -1$

$\Rightarrow x = \pm \pi/4 ; -\pi/2 \le x \le \pi/2$

(i) $k < \frac{1}{4}$ [illegible] $k > 1$ [illegible]

$y = \cos^6 x + \sin^6 x$ [illegible] $y = k$ [illegible] $\cos^6 x + \sin^6 x = k$ [illegible]

(ii) $k = \frac{1}{4}$ [illegible] $y = k$ [illegible]

$y = \cos^6 x + \sin^6 x$ [illegible]

$x = -\pi/4$ [illegible] $x = \pi/4$ [illegible]

$\cos^6 x + \sin^6 x = k$ [illegible]

(iii) $k = 1$ [illegible] $y = k$ [illegible] $y = \cos^6 x + \sin^6 x$ [illegible] $x = -\pi/2, 0$ [illegible] $\pi/2$ [illegible] $\cos^6 x + \sin^6 x = k$ [illegible]

(vi) $\frac{1}{4} < k < 1$ [illegible] $y = \cos^6 x + \sin^6 x$

[illegible] $y = k$ [illegible]

[illegible] $\cos^6 x + \sin^6 x = k$ [illegible]

(c) $4\sin^2 x + 12\sin x\cos x - \cos^2 x + 5 = 0$

$2(1 - \cos 2x) + 6\sin 2x - \frac{\cos 2x + 1}{2} + 5 = 0$

$\Rightarrow 12\sin 2x - 5\cos 2x + 13 = 0$

$\frac{12}{13}\sin 2x - \frac{5}{13}\cos 2x = -1$

$\frac{5}{13}\cos 2x - \frac{12}{13}\sin 2x = 1$

$\cos\alpha = 5/13$, $\sin\alpha = 12/13$ [illegible]
$\cos\alpha, \sin\alpha > 0$ [illegible] $0 < \alpha < \pi/2$

[illegible] $\cos\alpha\cos 2x - \sin\alpha\sin 2x = 1$

[illegible] $\cos(2x + \alpha) = 1$

$2x + \alpha = 2n\pi \quad n \in \mathbb{Z}$

[illegible] $x = n\pi - \alpha/2$

$n = 1$ [illegible] $x = \pi - \alpha/2$ [illegible]

$0 < \pi - \alpha/2 < 2\pi$

$n = 2$ [illegible] $x = 2\pi - \alpha/2$ [illegible]

$0 < 2\pi - \alpha/2 < 2\pi$

[illegible] $x = \pi - \alpha/2$ [illegible] $x = 2\pi - \alpha/2$ [illegible] $0 < x < 2\pi$ [illegible]

(d) [illegible] $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

[illegible] $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\frac{b + c}{2k - 1} = \frac{c + a}{2k} = \frac{a + b}{2k + 1} = \lambda$

[illegible]

$b + c = \lambda(2k - 1)$ — ①
$c + a = 2\lambda k$ — ②
$a + b = \lambda(2k + 1)$ — ③

① ② [illegible]
$b - a = -\lambda$ — ④

③ ② [illegible]
$c - b = -\lambda$ — ⑤

④ + ⑤
$c - a = -2\lambda$ — ⑥

$c + a = 2\lambda k \quad \therefore 2c = 2\lambda(k - 1)$

$\frac{c}{k - 1} = \lambda$

③ + ④

$2b = 2\lambda k \Rightarrow \frac{b}{k} = \lambda$

② - ⑥

$2a = 2\lambda k + 2\lambda \Rightarrow \frac{a}{1 + k} = \lambda$

[illegible] $\frac{a}{k + 1} = \frac{b}{k} = \frac{c}{k - 1}$

$\Rightarrow \frac{\sin A}{k + 1} = \frac{\sin B}{k} = \frac{\sin C}{k - 1}$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$= \frac{k^2 + (k - 1)^2 - (k + 1)^2}{2k(k - 1)}$

$= \frac{k - 4}{2(k - 1)}$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$= \frac{(k + 1)^2 + (k - 1)^2 - k^2}{2(k + 1)(k - 1)}$

$= \frac{k^2 + 2}{2(k^2 - 1)}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{(k + 1)^2 + k^2 - (k - 1)^2}{2k(k + 1)}$

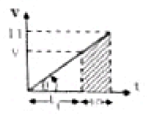
$= \frac{k + 4}{2(k + 1)}$

$\frac{\cos A}{(k - 4)(k + 1)} = \frac{\cos B}{k^2 + 2}$

$= \frac{\cos C}{(k + 4)(k - 1)}$ //

01. (a) [illegible]

[illegible] B [illegible] V [illegible] B [illegible] 11 ms^{-1} [illegible] 10 [illegible] 100 m [illegible]



$\therefore \frac{1}{2}(V + 11)\,10 = 100 \Rightarrow V = 9$

[illegible] B [illegible] t_1 [illegible]

$\tan\theta = \frac{V}{t_1} = \frac{11 - V}{10} \Rightarrow t_1 = 45$

(V = 9 [illegible])

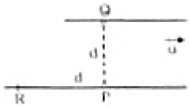
[illegible] $= \frac{9}{45} = \frac{1}{5}\ ms^{-2}$

[illegible] (45 + 10) [illegible]

$= \frac{1}{2} \times 55 \times 11$

$= \underline{302.5\ m}$

(b)



(B [illegible]) = v

(E [illegible]) = u

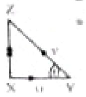
(B E) = [illegible]

(B E) = (B [illegible]) + ([illegible] E)

$= \underset{\rightarrow}{v} + \underset{\rightarrow}{u} = \underset{\rightarrow}{u} + \underset{\rightarrow}{v}$

$= \overrightarrow{XY} + \overrightarrow{YZ}$

$= \overrightarrow{XZ}$



$XZ = \sqrt{v^2 - u^2}$

$\therefore$ P [illegible] Q [illegible] $T = \frac{d}{\sqrt{v^2 - u^2}}$

PQ = PR = d [illegible]

[illegible] $= v - u$

[illegible] $= v + u$

$P \rightarrow R \rightarrow P$ [illegible]

$T_1 = \frac{d}{v - u} + \frac{d}{v + u}$

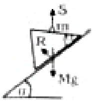
$= \frac{dv + du + dv - du}{(v - u)(v + u)}$

$= \frac{2dv}{v^2 - u^2} = \frac{d}{\sqrt{v^2 - u^2}} \times \frac{2v}{\sqrt{v^2 - u^2}}$ [illegible]

$T_1 = \frac{2Tv}{\sqrt{v^2 - u^2}}$

v > u [illegible]

02. (a) [illegible]



(m, [illegible]) = $\overleftarrow{a_1}$

(M E) = a_2 [illegible]

(m, E) = (m, [illegible]) + (M E)

$= \overleftarrow{a_1} + a_2$ [illegible]

$\underline{F} = m\,\underline{a}$

(m) $\leftarrow$ 0 $= m(a_1 - a_2\cos\alpha)$ — ❶

(m+M) [illegible]

$(M + m)\,g\sin\alpha$

$= Ma_2 + m(a_2 - a_1\cos\alpha)$ — ❷

❶ [illegible] $a_1 = a_2\cos\alpha$; ❷ [illegible]

$(M + m)\,g\sin\alpha$

$= a_2\,[M + m - m\cos\alpha\cos\alpha]$

$a_2 = \frac{(M + m)g\sin\alpha}{M + m\sin^2\alpha}$

[illegible] (m E) = $\overrightarrow{a_1}$ + [illegible] a_2

[illegible] $= a_1 - a_2\cos\alpha$

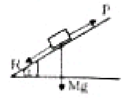
$= 0$ ❶ [illegible]

[illegible] $= \downarrow a_2\sin\alpha$

[illegible]
$[(M + m)g\sin^2\alpha]/[M + m\sin^2\alpha]$ [illegible]

(b) [illegible] $= 24\ km\,h^{-1} = \frac{20}{3}\ ms^{-1}$ [illegible]

[illegible] P [illegible] $H = 500 \times 10^3\ W$
[illegible] PV = H [illegible]



$P = \frac{500 \times 10^3}{20/3} = 75000\ N$ [illegible]

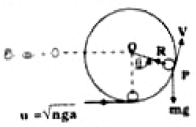
[illegible] R N [illegible] $\underline{F} = m\underline{a}$ [illegible]

$75000 - R - 2.5 \times 10^5 \times \frac{1}{196} \times 9.8$

$= 2.5 \times 10^5 \times 0.2$

$R = 12500\ N$ [illegible]

03. [illegible]



[illegible]

$\frac{1}{2}m\ nga - mga = \frac{1}{2}mV^2 - mga\cos\theta$

$V^2 = ga\,(n - 2 + 2\cos\theta)$ — ❶

[illegible] $\underline{F} = m\,\underline{a}$ [illegible]

$R - mg\cos\theta = m\frac{V^2}{a}$ — ❷

❶ [illegible]

$R = mg\cos\theta + \frac{m}{a}ga\,(n - 2 + 2\cos\theta)$

$R = mg\,(n - 2 + 3\cos\theta)$

[illegible] R = 0 [illegible]

$R = 0 \Rightarrow \cos\alpha = \frac{2 - n}{3}$; [illegible] θ = α [illegible]

$2 < n < 5$ [illegible] $\alpha > \pi/2$ [illegible]

($\because \cos\alpha < 0$ [illegible])

[illegible] ❶ [illegible]

$V_0^2 = ga\left[n - 2 + 2\,\frac{(2 - n)}{3}\right]$

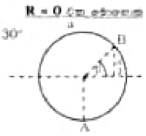
$= ga\left[\frac{3n - 6 + 4 - 2n}{3}\right]$

$= ga\,\frac{(n - 2)}{3}$

[illegible] V_0 [illegible] θ = α [illegible]

(ii) O [illegible] $\frac{a}{2}$ [illegible]

R = 0 [illegible]



$\sin\beta = \frac{1}{2}$ [illegible] $\Rightarrow \beta = 30°$

$\alpha = \frac{\pi}{2} + \beta$

[illegible] $\cos\alpha = -\frac{1}{2}$

$\cos\alpha = \frac{2 - n}{3} = -\frac{1}{2} \Rightarrow 4 - 2n = -3$

$\Rightarrow \underline{n = \frac{7}{2}}$

t_1 [illegible] $V_0^2 = \frac{ga}{3}\left[\frac{7}{2} - 2\right] = \frac{ga}{2}$ [illegible]

[illegible] $a\cos 30 = \frac{a\sqrt{3}}{2}$ [illegible] t_1 [illegible]

$S = ut + \frac{1}{2}at^2$ [illegible] $\frac{a\sqrt{3}}{2} =$

$\sqrt{\frac{ga}{2}}\cos 60\ t_1 \Rightarrow t_1 = \sqrt{\frac{6a}{g}}$ [illegible]

t_1 [illegible] h [illegible]

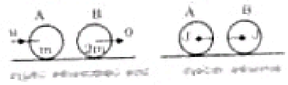
$h = \sqrt{\frac{ga}{2}}\sin 60 \cdot t_1 - \frac{1}{2}g \cdot t_1^2$

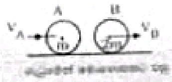
$h = \sqrt{\frac{ga}{2}} \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{\frac{6a}{g}} - \frac{1}{2}g \cdot \frac{6a}{g}$

$= \frac{3}{2}a - 3a = -\frac{3a}{2}$

[illegible] B [illegible] A [illegible]
[illegible] A [illegible]

04. (a) [illegible]





(i) [illegible]

$mu = 2m\,V_B + m\,V_A$ — ❶

[illegible]

$eu = V_B - V_A$ — ❶

❶+❷ න් $V_B = \frac{u}{3}(1+e)$; ❶-❷ × 2 න්

$V_A = \frac{u}{3}(1-2e)$

$I = \Delta(m\underline{V})$, (B) ට [illegible]

$J = 2m\,V_B = \frac{2mu}{3}(1+e)$

(A) ට [illegible] $J = m(-V_A) - m(-u)$

$= mu - mV_A$

$= m(u - V_A)$

(ii) [illegible] E = [illegible]

$E = \frac{1}{2}mu^2 - \left[\frac{1}{2}mV_A^2 + \frac{1}{2}\,2m\,V_B^2\right]$

$= \frac{1}{2}m\left[u^2 - V_A^2\right] - \frac{1}{2}\,2m\,V_B^2$

$= \frac{1}{2}m(u - V_A)(u + V_A) - \frac{1}{2}2m\,V_B.V_B$

$= \frac{1}{2}J(u + V_A) - \frac{1}{2}J\,V_B$

$= \frac{1}{2}Ju + \frac{1}{2}J(V_A - V_B)$

$= \frac{1}{2}Ju + \frac{1}{2}J(-eu)$

$= \frac{1}{2}J(1-e)u$

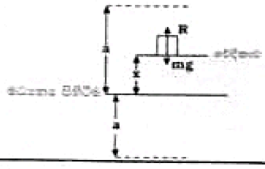
(iii) [illegible] A [illegible] $V_A < 0$ [illegible]

$1 - 2e < 0 \Rightarrow e > \frac{1}{2}$ [illegible]

$E = \frac{1}{2}\,\frac{2mu}{3}(1+e)(1-e)u$

$= \frac{mu^2}{3}(1-e^2) < \frac{mu^2}{3}\left[1 - \left(\frac{1}{2}\right)^2\right]$

$E < \frac{mu^2}{3}\left(1 - \frac{1}{4}\right) \Rightarrow E < \frac{1}{4}mu^2$



[illegible]

$E = mg\uparrow$

$R - mg = m\ddot{x}$ — ❶

[illegible]

$\ddot{x} = -\omega^2 x$ — ❷

[illegible] T [illegible] $T = \frac{2\pi}{\omega}$

❷ න් $\ddot{x} = -\frac{4\pi^2}{T^2}x$

❶ ට [illegible]

$R = mg - m\frac{4\pi^2 x}{T^2} = m\left[g - \frac{4\pi^2 x}{T^2}\right]$

R [illegible] x [illegible]
[illegible] x = [illegible] = a, [illegible] $R \geq 0$ [illegible]

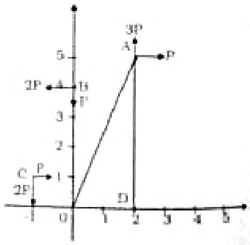
$m\left[g - \frac{4\pi^2 a}{T^2}\right] \geq 0$ [illegible]

$\pi^2 = g = 9.8$ [illegible] $T = 1$ [illegible]

$9.8\,m\left[1 - \frac{4a}{T}\right] \geq 0$ [illegible] $a \leq 0.25$ m [illegible]

∴ [illegible] 0.25 m [illegible]

05. [illegible]



$\overrightarrow{X} = P + P - 2P = 0$

$\uparrow Y = 3P - P - 2P = 0$

∴ $R = 0$ [illegible]

O [illegible] G [illegible]

$G = [(2P\times4) + (2P\times1) + (3P\times2) - (P\times5) - (P\times1)]$ Nm

$= 10$ P Nm

[illegible] ABC [illegible]

OAD [illegible] $\lambda\overrightarrow{OA}$, $\lambda\overrightarrow{AD}$, $\lambda\overrightarrow{DO}$ [illegible]

$\lambda\overrightarrow{OA} + \lambda\overrightarrow{AD} = \lambda\overrightarrow{OD}$. [illegible] A [illegible]

$\lambda\overrightarrow{OA} + \lambda\overrightarrow{AD} + \lambda\overrightarrow{DO}$

$=$ A [illegible] $\lambda\overrightarrow{OD}$ + D [illegible] $\lambda\overrightarrow{DO}$

[illegible] ADO [illegible] $\underline{j}$ 5 × 2λ $\underline{j}$ [illegible]

[illegible]

$10P - 10\lambda = 0 \Rightarrow \lambda = P$

OA [illegible] $= P\overrightarrow{OA}$

$= P(2\underline{i} + 5\underline{j})$

AD [illegible] $= P\overrightarrow{AD} = P(-5\underline{j})$

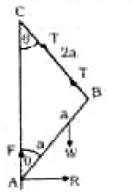
$= -5P\underline{j}$

DO [illegible] $= P\overrightarrow{DO} = P(-2\underline{j})$

$= -2P\underline{j}$

06. [illegible]

(a) [illegible]
[illegible] $T\sin\theta = R$ — ❶
[illegible] $F + T\cos\theta = W$ — ❷



A [illegible]
$T \times 2a\sin 2\theta = W.\,a\sin\theta$

$T = \frac{W}{4\cos\theta}$

[illegible] ❶ [illegible] $R = \frac{W}{4}\tan\theta$

❷ [illegible] $F = \frac{3}{4}W$

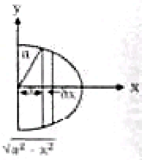
[illegible] $\frac{F}{R} \leq \mu$ [illegible]

[illegible] $\cot\theta \leq \frac{\mu}{3}$ [illegible]

[illegible] $\theta \leq \cot^{-1}\left(\frac{\mu}{3}\right)$

(b) [illegible] G [illegible] X [illegible] $G \equiv (\bar{x}, 0)$

Y [illegible] x [illegible] δx [illegible]



[illegible] $= \sqrt{a^2 - x^2}$

[illegible] ρ [illegible]

$= \pi(a^2 - x^2)\rho\,\delta x$

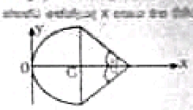
[illegible] Y [illegible] X [illegible]

$\bar{x} = \frac{\int_0^a \pi(a^2 - x^2)\rho x\,dx}{\int_0^a \pi(a^2 - x^2)\rho\,dx}$

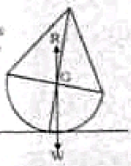
[illegible] $\bar{x} = \frac{3}{8}a$

∴ $G \equiv (\frac{3}{8}a, 0)$

[illegible]
[illegible] X [illegible]



[illegible]



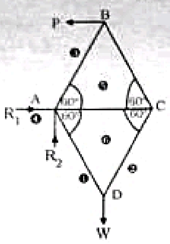
$\overline{G}$ [illegible]

$\frac{2}{3}\pi a^3\rho.\frac{3}{8}a$

$= \frac{1}{3}\pi a^2 a\cot\alpha\rho \times \frac{1}{4}a\cot\alpha$

$\cot^2\alpha = 3 \Rightarrow \tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = 30^\circ$

07. [illegible] A [illegible] R_1 [illegible] R_2 [illegible]



A [illegible]

$W \times a \cos 60^\circ = P \times a \sin 60^\circ \Rightarrow$

$P = W/\sqrt{3}$ [illegible]

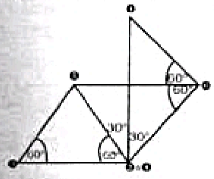
$\uparrow$ [illegible] : $R_2 = W \rightarrow R_1 = P = W/\sqrt{3}$

[illegible] $\overrightarrow{AC}$ [illegible] θ [illegible]

$\tan\theta = \frac{R_2}{R_1} = \frac{W}{W/\sqrt{3}} = \sqrt{3} \Rightarrow \theta = 60^\circ$

A [illegible] AB [illegible]

[illegible]	[illegible]	[illegible]
AD	[illegible]	$W/\sqrt{3}$
CD	[illegible]	$W/\sqrt{3}$
BC	[illegible]	$W/\sqrt{3}$
AB	[illegible]	$W/\sqrt{3}$
AC	[illegible]	$W/\sqrt{3}$



08. (අ) [illegible]

(ආ) [illegible]

$P(A \cap B) = \frac{1}{4}$; $P(A) = \frac{5}{12}$;

$P(A | B') = \frac{5}{12}$

(i) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{4}}{\frac{5}{12}} = \frac{3}{5}$

(ii) $P(A|B') = \frac{P(A \cap B')}{P(B')} \Rightarrow P(A \cap B')$

$= P(A|B')\, P(B')$

$P(A \cap B') = \frac{5}{12} P(B')$ — ❶

$P(A) = P(A \cap B) + P(A \cap B')$ [illegible]

$\frac{5}{12} = \frac{1}{4} + P(A \cap B') \Rightarrow P(A \cap B')$

$= \frac{1}{6}$

❶ [illegible] $\frac{1}{6} = \frac{5}{12} P(B') \Rightarrow P(B') = \frac{2}{5}$

$P(B) = 1 - P(B') \Rightarrow P(B) = \frac{3}{5}$

(iii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A|B) = \frac{\frac{1}{4}}{\frac{3}{5}} = \frac{5}{12}$

(iv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{5}{12} + \frac{3}{5} - \frac{1}{4} = \frac{23}{30}$

$P(A \cap B) = \frac{1}{4} \neq 0$ [illegible] $A \cap B \neq \phi$ [illegible]

∴ A හා B [illegible]

$P(A)\,P(B) = \frac{5}{12} \times \frac{3}{5} = \frac{1}{4}$

$= P(A \cap B)$ [illegible]

A හා B [illegible]

(ආ) B - [illegible]

G - [illegible]

B හා G [illegible]

$P(B) = P(G) = \frac{1}{2}$ [illegible]

A [illegible] $= P(B|A)$

(i) $P(B|A) = \frac{P(B \cap A)}{P(A)}$ — ❶

$P(A)$

$= P(BB) + P(BG) + P(GB)$

$= P(B)P(B) + P(B)P(G) + P(G)P(B)$

$= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{3}{4}$

$P(B \cap A)$ [illegible]

$\therefore P(B \cap A) = \frac{1}{4}$

❶ [illegible] $P(B|A) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

(ii) $P(G|A) = 1 - P(B|A)$

$= 1 - \frac{1}{3} = \frac{2}{3}$

(G | A හා B | A [illegible])

09. [illegible]

[illegible]	[illegible]	$\frac{x-72}{5} = d$	f	fd	fd^2	[illegible]
50-54	52	-4	3	-12	48	3
55-59	57	-3	7	-21	63	10
60-64	62	-2	15	-30	60	25
65-69	67	-1	38	-38	38	63
70-74	72	0	41	0	0	104
75-79	77	1	35	35	35	139
80-84	82	2	21	42	84	160
85-89	87	3	16	48	144	176
90-94	92	4	14	56	224	190
95-99	97	5	10	50	250	200
			200	130	946	

(අ) (i) [illegible] 100 [illegible]

[illegible] 70 - 74 [illegible] 69.5 - 74.5 [illegible]

[illegible] $= L + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times i$

(C. F. [illegible])

$= 69.5 + \frac{\frac{200}{2} - 63}{41} \times 5$

$= 74.01 \triangleq 74.0$

(ii) [illegible] Q_1 [illegible]

[illegible] 65 - 69 [illegible] 64.5 - 69.5

$Q_1 = 64.5 + \left(\frac{\frac{200}{4} - 25}{38}\right) \times 5$

$= 67.79 \triangleq 67.8$

(iii) [illegible] 80 - 84 [illegible] 79.5 - 84.5

$Q_3 = 79.5 + \frac{\frac{200}{4} \times 3 - 139}{21} \times 5$

$= 82.12 \triangleq 82.1$

(ආ) [illegible]

(i) [illegible] $\bar{x}$

$=$ [illegible] $+ \frac{i \sum fd}{\sum f}$

[illegible] $i = 5$

$\bar{x} = 72 + 5 \times \frac{130}{200}$

$= 75.25 \triangleq 75.3$

(ii) [illegible] $= \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$

$= \sqrt{\frac{946}{200} - \left(\frac{130}{200}\right)^2} \times 5$

$= 10.377 \triangleq 10.4$

(iii) [illegible]

$= \frac{3(\text{[illegible]} - \text{[illegible]})}{\text{[illegible]}}$

$= \frac{3(75.25 - 74)}{10.377} \triangleq 0.4$

[illegible]

❖❖❖ ❖❖❖

Past Papers Wiki - Most Extensive Wikipedia of Past Papers!