01. (i) $P(x) = (\lambda - 2)x^2 - 3(\lambda + 2)x + 6\lambda$

$\lambda = 2$ විට $P(x) = -12x + 12$ වේ.

$\lambda = 2$ හා $x = 3$ විට $P(3) = -24 < 0$

එනම් $\lambda = 2$ විට සියලු තාත්වික x සඳහා p(x) ධන නොවේ.

$\lambda \neq 2$ විට $\lambda - 2 < 0$ විට සියලු x සඳහා p(x) ධන වන λ අගයක් නොපවතී.

$\lambda - 2 > 0$ හා

$\Delta_x = 9(\lambda + 2)^2 - 4(\lambda - 2)6\lambda < 0$ විට

පමණක් $\forall x \in \mathbb{R}$ සඳහා $P(x) > 0$ වේ.

එනම් $\lambda > 2$ හා $\Delta = 9(\lambda^2 + 4\lambda + 4)$
$- 24\lambda^2 + 48\lambda < 0$

එනම් $\lambda > 2$ හා

$-15\lambda^2 + 84\lambda + 36 < 0$

$\Rightarrow \lambda > 2$ හා $5\lambda^2 - 28\lambda - 12 > 0$

$\Rightarrow \lambda > 2$ හා $(5\lambda + 2)(\lambda - 6) > 0$

$\Rightarrow \lambda > 2$ හා $\lambda < -\frac{2}{5}$ හෝ $\lambda > 6$

$\Rightarrow \lambda > 6$

$\therefore$ x හි සියලු අගය සඳහා $P(x) > 0$ වන λ හි කුඩාතම නිඛිල අගය $\lambda = 7$ වේ.

(ii) P(x) ට ප්‍රභින්න තාත්වික මූල තිබීමට නම් $\lambda \neq 2$ හා $\Delta > 0$ විය යුතුය.

එනම් $\lambda \neq 2$ හා $(5\lambda + 2)(\lambda - 6) < 0$

$\Rightarrow \lambda \neq 2$ හා $\frac{-2}{5} < \lambda < 6$

එනම් $\lambda \in \left(\frac{-2}{5}, 2\right) \cup (2, 6)$

(iii) $P(x) = 0$ හි තාත්වික ප්‍රභින්න මූල α හා β යැයි ද $\alpha > \beta$ යැයි ද සිතමු.

එවිට $\alpha + \beta = \frac{3(\lambda + 2)}{(\lambda - 2)}$ හා $\alpha\beta = \frac{6\lambda}{\lambda - 2}$ වේ.

$\alpha - \beta = 3$ නම්

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$9 = \frac{9(\lambda + 2)^2}{(\lambda - 2)^2} - \frac{24\lambda}{\lambda - 2}$

$\Rightarrow 9(\lambda - 2)^2 = 9(\lambda + 2)^2 - 24\lambda(\lambda - 2)$

$\Rightarrow 9\lambda^2 - 36\lambda + 36$

$= 9\lambda^2 + 36\lambda + 36 - 24\lambda^2 + 48\lambda$

$\Rightarrow 24\lambda^2 - 120\lambda = 0$

$\Rightarrow \lambda^2 - 5\lambda = 0$

$\Rightarrow \lambda(\lambda - 5) = 0$

$\therefore \lambda = 0$ හෝ $\lambda = 5$

02. (a)

සංයෝජන වර්ග		සංයෝජන ගණන	
(a)	5, 1, 1, 1	$\frac{8!}{5!(1!)^3 3!}$	$= 56$
(b)	4, 2, 1, 1	$\frac{8!}{4!2!(1!)^2 2!}$	$= 420$
(c)	3, 3, 1, 1	$\frac{8!}{(3!)^2(1!)^2 2!2!}$	$= 280$
(d)	3, 2, 2, 1	$\frac{8!}{3!(2!)^2 1!2!}$	$= 840$
(e)	2, 2, 2, 2	$\frac{8!}{(2!)^4 4!}$	$= 105$

$\therefore$ තිබිය හැකි මුළු සංයෝජන ගණන

$= 56 + 420 + 280 + 840 + 105$

$= \underline{1701}$

(b) ${}^nC_{r+1} + {}^nC_r$

$= \frac{n!}{(n - r - 1)!(r + 1)!} + \frac{n!}{(n - r)!r!}$

$= \frac{n!(n - r)}{(n - r)!(r + 1)!} + \frac{n!(r + 1)}{(n - r)!(r + 1)!}$

$= \frac{n!(n - r + r + 1)}{(n - r)!(r + 1)!}$

$= \frac{(n + 1)!}{(n + 1 - r + 1)!(r + 1)!}$

$= {}^{n+1}C_{r+1}$

ඉහත ප්‍රතිඵලය අනුව

${}^{2003}C_r = {}^{2004}C_{r+1} - {}^{2003}C_{r+1}$

${}^{2004}C_r = {}^{2005}C_{r+1} - {}^{2004}C_{r+1}$

${}^{2005}C_r = {}^{2006}C_{r+1} - {}^{2005}C_{r+1}$

$\vdots$

${}^{2013}C_r = {}^{2014}C_{r+1} - {}^{2013}C_{r+1}$

${}^{2003}C_r + {}^{2004}C_r + \ldots {}^{2013}C_r$

$= \underline{{}^{2014}C_{r+1} - {}^{2003}C_{r+1}}$

03. (a) $F(n) : 8(n + 1)! > 2^{n+1}(n + 2)$ යැයි ගනිමු.

$n = 1$ විට

L.H.S $= 8 \times 2! = 16$

R.H.S $= 4 \times 3 = 12$

$\therefore P(1)$ [illegible]

$P(p)$ [illegible] $p \in \mathbb{Z}^+$ [illegible]

[illegible] $8(p+1)! > 2^{p+1}(P+2)$

[illegible] $(p+2)$ [illegible]

[illegible]

$8(P+1)!(P+2) > 2^{P+1}(P+2)^2$

$\Rightarrow 8(p+2)! > 2^{P+1}(p+2)^2$

$= 2^{P+1}(p^2+4p+4)$

$= 2^{P+1}[(2p+3)+(p+1)^2]$

$> 2^{P+1}[(2p+3)+3]$

$\forall p \in \mathbb{Z}, (p+1)^2 > 3$

[illegible] $8(p+2)! > 2^{p+2}(p+3)$

$\Rightarrow 8(\overline{p+1}+1)! > 2^{\overline{p+1}+1}(\overline{p+1}+2)$

$\therefore P(p+1)$ [illegible]

[illegible] n [illegible] $P(n)$ [illegible]

[illegible]

$\frac{(n+1)!}{2^{n+1}} > \frac{(n+2)}{8}$ [illegible] $n = 1, 2, 3 \ldots$

$\therefore \sum_{k=1}^{n} \frac{k!}{2^k}$

$= \frac{1}{2} + \sum_{k=1}^{n-1} \frac{(k+1)!}{2^{k+1}} > \frac{1}{2} + \sum_{k=1}^{n-1} \frac{(k+2)}{8}$

$= \frac{1}{2} + \frac{1}{8} \sum_{k=1}^{n-1} (k+2)$

$= \frac{1}{2} + \frac{1}{8} \frac{(n-1)n}{2} + \frac{2(n-1)}{8}$

$= \frac{1}{16}(n^2+3n+4)$

$S_n = \sum_{k=1}^{n} \frac{k!}{2^k} > \frac{1}{16}(n^2+3n+4);\ \forall n \in \mathbb{Z}^+$

$\lim_{n\to\infty} S_n > \lim_{n\to\infty} \frac{1}{16}(n^2+3n+4) = \infty$

$\Rightarrow \lim_{n\to\infty} S_n = \infty$

$\therefore$ [illegible]

(b) $x < -2$ [illegible]

$-(x+2) + \{-(x-1)\} > 5$

$= -x - 2 - x + 1 = -2x - 1 > 5$

$\Rightarrow x < -3$ [illegible] $x < -3$ [illegible]

$-2 \le x < 1$ [illegible]

$(x+2) + \{-(x-1)\} = x + 2 - x + 1 = 3 > 5$

[illegible] $\therefore -2 \le x < 1$ [illegible]

$1 \le x$ [illegible]

$(x+2) + (x-1) = 2x + 1 > 5$

$\Rightarrow x > 2$

[illegible] $x > 2$ [illegible]

$\therefore$ [illegible] x [illegible]

$\{x \in \mathbb{R} : x < -3\} \cup \{x \in \mathbb{R} : x > 2\}$

[illegible] $x \in (-\infty, -3) \cup (2, \infty)$

04. $\omega = \sqrt{3} + i = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$\omega^2 = \omega\omega$

$= 2^2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= 4\left[\left(\cos^2\frac{\pi}{6} - \sin^2\frac{\pi}{6}\right) + i\left(2\sin\frac{\pi}{6}\cos\frac{\pi}{6}\right)\right]$

$= 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\omega^3 = \omega^2\omega$

$= 8\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= 8\left[\left(\cos\frac{\pi}{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{3}\sin\frac{\pi}{6}\right) + i\left(\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6}\right)\right]$

$= 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$\omega^4 = \omega^3\omega$

$= 16\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

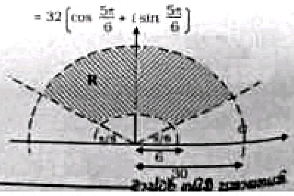
$= 16\left[\left(\cos\frac{\pi}{2}\cos\frac{\pi}{6} - \sin\frac{\pi}{2}\sin\frac{\pi}{6}\right) + i\left(\sin\frac{\pi}{2}\cos\frac{\pi}{6} + \cos\frac{\pi}{2}\sin\frac{\pi}{6}\right)\right]$

$= 16\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$\omega^5 = \omega^4\omega$

$= 32\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= 32\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$



$|\omega| = 2 < 6$

$|\omega^2| = 4 < 6$ [illegible] $|\omega^5| = 32 > 30$

[illegible] ω, ω^2 [illegible] ω^5 R [illegible]

$|\omega^3| = 8 \Rightarrow \arg(\omega^3) = \frac{\pi}{2}$ [illegible]

$|\omega^4| = 16 \Rightarrow \arg(\omega^4) = \frac{2\pi}{3}$

[illegible] ω^3 [illegible] ω^4 [illegible] R [illegible]

05. (a) $y = e^{\cos x}$

$\Rightarrow \frac{dy}{dx} = e^{\cos x}(-\sin x)$

$\left(\frac{dy}{dx}\right)_{x=0} = 0$

$\left(\frac{d^2y}{dx^2}\right) = e^{\cos x}(-\cos x) + (-\sin x)e^{\cos x}(-\sin x)$

$= -e^{\cos x}\cos x + \sin^2 x e^{\cos x}$

$\left(\frac{d^2y}{d^2x}\right)_{x=0} = -e$

$\frac{dy}{dx} = -y\sin x$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx}\sin x - y\cos x$

$\left(\frac{d^3y}{dx^3}\right) = -\frac{d^2y}{dx^2}\sin x - \frac{dy}{dx}\cos x - \frac{dy}{dx}\cos x + y\sin x$

$\left(\frac{d^3y}{dx^3}\right)_{x=0} = 0$

$\frac{d^3y}{dx^3} = -\left(\frac{d^2y}{dx^2}\right)\sin x - 2\frac{dy}{dx}\cos x - \frac{dy}{dx}$

$\frac{d^4y}{dx^4} = -\left(\frac{d^3y}{dx^3}\right)\sin x - \left(\frac{d^2y}{dx^2}\right)\cos x - 2\left(\frac{d^2y}{dx^2}\right)\cos x + 2\frac{dy}{dx}\sin x - \frac{d^2y}{dx^2}$

$\left(\frac{d^4y}{dx^4}\right)_{x=0} = 4e$

$\frac{d^4y}{dx^4} = -\left(\frac{d^3y}{dx^3}\right)\sin x - 3\left(\frac{d^2y}{dx^2}\right)\cos x - \frac{d^2y}{dx^2} + 2\frac{dy}{dx}\sin x$

$\frac{d^5y}{dx^5} = -\left(\frac{d^4y}{dx^4}\right)\sin x - \left(\frac{d^3y}{dx^3}\right)\cos x - 3\left(\frac{d^3y}{dx^3}\right)\cos x + 3\left(\frac{d^2y}{dx^2}\right)\sin x - \left(\frac{d^3y}{dx^3}\right) + 2\left(\frac{d^2y}{dx^2}\right)\sin x + 2\frac{dy}{dx}\cos x$

$= -\left(\frac{d^4y}{dx^4}\right)\sin x - 4\left(\frac{d^3y}{dx^3}\right)\cos x - \left(\frac{d^3y}{dx^3}\right) + 5\left(\frac{d^2y}{dx^2}\right)\sin x + 2\frac{dy}{dx}\cos x$

$\left(\frac{d^5y}{dx^5}\right)_{x=0} = 0$

(b) $y = \frac{2x}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$

$= \frac{2(1-x^2)}{(1+x^2)^2}$

$= \frac{2(1+x)(1-x)}{(1+x^2)^2}$

$\frac{dy}{dx} = 0 \Rightarrow x = \pm 1$

$x = 1$ [illegible] $y = 1$ [illegible]

$x = -1$ [illegible] $y = -1$

[illegible]	$\frac{dy}{dx}$	
$x < -1$	(-)	[illegible]
$x = -1$	0	
$-1 < x < 1$	(+)	
$x = 1$	0	[illegible]
$1 < x$	(-)	

$\therefore (-1, -1)$ [illegible]

$(1, 1)$ [illegible]

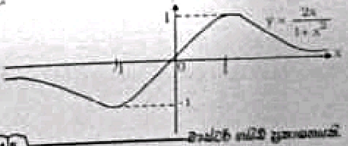
$x = 0$ [illegible] $y = 0$ [illegible]

$y = 0$ [illegible] $x = 0$ [illegible]

$y = \frac{2/x}{1/x^2 + 1}$

$x \to -\infty$ [illegible] $y \to 0^-$

$x \to \infty$ [illegible] $y \to 0^+$



06. (a) $\sqrt[3]{x} = y$

$x = y^3 \Rightarrow dx = 3y^2 dy$

$\therefore \int_1^8 \frac{dx}{1 + x^{1/3}}$

$= \int_1^2 \frac{3y^2 dy}{1 + y}$

$= 3\int_1^2 \left[(y - 1) + \frac{1}{1 + y}\right] dy$

$= 3\left[\frac{y^2}{2} - y + \ln|1 + y|\right]_1^2$

$= 3\left[\ln 3 - \frac{1}{2} + 1 - \ln 2\right]$

$= 3\left[\frac{1}{2} + \ln\frac{3}{2}\right]$

(b) $\int_0^1 x^2 e^{2x + 3}\, dx$

$= e^3 \int_0^1 x^2 e^{2x} dx$

$= e^3 \int_0^1 x^2 \frac{d}{dx}\left(\frac{e^{2x}}{2}\right) dx$

$= e^3 \left[\left(x^2 \frac{e^{2x}}{2}\right)_0^1 - \int_0^1 \frac{e^{2x}}{2} 2x dx\right]$

$= e^3 \left[\frac{e^2}{2} - \int_0^1 x \frac{d}{dx}\left(\frac{e^{2x}}{2}\right) dx\right]$

$= e^3 \left[\frac{e^2}{2} - \left(x \frac{e^{2x}}{2}\right)_0^1 + \int_0^1 \frac{e^{2x}}{2} dx\right]$

$= e^3 \left[\frac{e^2}{2} - \frac{e^2}{2} + \left(\frac{e^{2x}}{4}\right)_0^1\right]$

$= e^3\left(\frac{e^2}{4} - \frac{1}{4}\right) = \frac{e^3}{4}(e^2 - 1)$

(c) $\frac{1}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$

$\Rightarrow A = \frac{1}{3}, \quad B = \frac{-1}{3}$ සහ $C = 0$

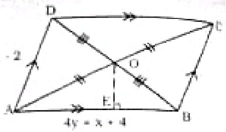
$\int \frac{dx}{x(x^2 + 3)} = \int \frac{dx}{3x} - \int \frac{x}{3(x^2 + 3)} dx$

$= \frac{1}{3}\left(\ln|x| - \frac{1}{2}\ln|x^2 + 3|\right) + D$

$= \frac{1}{3}\ln\frac{|x|}{\sqrt{x^2 + 3}} + D$

D අභිමත නියතයකි.

07.



(i) $y = x - 2$ ——— ①
$4y = x + 4$ ——— ②

① හා ② න් $4(x - 2) = x + 4$

$\Rightarrow x = 4, y = 2$

$A \equiv (4, 2)$ එමෙන්ම $C \equiv (-4, -2)$

BC හි සමීකරණය $y = x + c$ ආකාර වේ.

$(-4, -2)$

$-2 = -4 + C \Rightarrow C = 2$

$\therefore$ BC හි සමීකරණය $y = x + 2$

CD හි සමීකරණය $4y = x + d$ ආකාර වේ.

$4(-2) = -4 + d \Rightarrow d = -4$

$4y = x - 4$

(ii) AC හි සමීකරණය

$\frac{y - 0}{x - 0} = \frac{-2 - 2}{-4 - 4} = \frac{1}{2} \Rightarrow 2y = x$

$4y - x - 4 + \lambda(y - x - 2) = 0$

$\lambda \in \mathbb{R}$

O $\lambda = -2$

$\therefore$ BD හි සමීකරණය

$4y - x - 4 - 2(y - x - 2) = 0$

$2y + x = 0$

AB $\Rightarrow$ $4y = x + 4$ ——— ③

BC $\Rightarrow$ $y = x + 2$ ——— ④

③ හා ④ න්

$B \equiv \left(\frac{-4}{3}, \frac{2}{3}\right)$

$= 4 \times$ AOB Δ

$= 4 \times \frac{1}{2}$ AB.OE

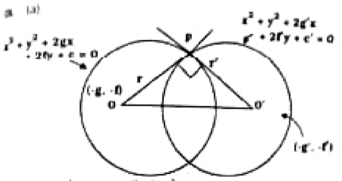
$= 4 \times \frac{1}{2} \times \sqrt{\left(4 + \frac{4}{3}\right)^2 + \left(2 - \frac{2}{3}\right)^2} \times \frac{4}{\sqrt{17}}$

$= 2 \times \sqrt{\frac{256}{9} + \frac{16}{9}} \times \frac{4}{\sqrt{17}}$

$= 2 \times \frac{4}{\sqrt{17}} \times \frac{4\sqrt{17}}{3}$

$= \frac{32}{3}$

08. (a)



$(PO)^2 + (PO')^2 = (OO')^2$

$(r)^2 + (r')^2 = (-g + g')^2 + (-f + f')^2$

$r = \sqrt{g^2 + f^2 - c}$

$r' = \sqrt{g'^2 + f'^2 - c'}$

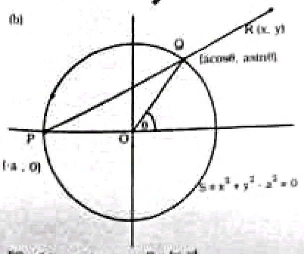
$(g^2 + f^2 - c) + (g'^2 + f'^2 - c')$

$= g^2 + g'^2 - 2gg' + f^2 + f'^2 - 2ff'$

$\Rightarrow -c - c' = -2gg' - 2ff'$

$\Rightarrow 2gg' + 2ff' = c + c'$

(b)



PQ : QR = 1:1 නම් R ≡ (x, y)

$\Rightarrow \frac{-a + x}{2} = a\cos\theta$ සහ $\frac{0 + y}{2} = a\sin\theta$

$\Rightarrow x = 2a\cos\theta + a,$

$y = 2a\sin\theta$

$\Rightarrow R \equiv (2a\cos\theta + a, 2a\sin\theta)$

$\cos\theta = \frac{x - a}{2a} \Rightarrow \sin\theta = \frac{y}{2a}$

$\cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \left(\frac{x - a}{2a}\right)^2 + \left(\frac{y}{2a}\right)^2 = 1$

$\Rightarrow (x - a)^2 + y^2 = 4a^2$

R, අරය 2a වූ (a, 0) ... S'

S' හි සමීකරණය $x^2 + y^2 - 2ax - 3a^2 = 0$

S'' හි සමීකරණය

$S'' \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$2g(0) + 2f(0) = c - a^2 \Rightarrow c = a^2$

$2g(-a) + 2f(0) = c - 3a^2$

$\Rightarrow -2ga = c - 3a^2$

$\Rightarrow -2ga = -2a^2 \Rightarrow g = a$

$\therefore S'' \equiv x^2 + y^2 + 2ax + 2fy + a^2 = 0$

$x = 0$ ... y

$y^2 + 2fy + a^2 = 0$

$4f^2 - 4a^2 = 0 \Rightarrow f^2 = a^2 \Rightarrow f = \pm a$

$S'' \equiv x^2 + y^2 + 2ax + 2ay + a^2 = 0$ හා

$S'' \equiv x^2 + y^2 + 2ax - 2ay + a^2 = 0$ වේ.

09. (a) (i) $x = \sin\theta - \cos\theta$

$\Rightarrow x^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$= 1 - \sin 2\theta$

$\Rightarrow \sin 2\theta = 1 - x^2$

(ii) $y = \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\frac{1}{2}\sin 2\theta} = \frac{2}{\sin 2\theta}$

$\Rightarrow \sin 2\theta = \frac{2}{y}$

$\therefore 1 - x^2 = \frac{2}{y} \Rightarrow y(1 - x^2) - 2 = 0$

(b) $\sin 2x + \sin 4x + \sin 6x$

$= \sin 4x + (\sin 2x + \sin 6x)$

$= \sin 4x + 2\sin 4x \cos 2x$

$= (1 + 2\cos 2x)\sin 4x$

$\sin x(\sin 2x + \sin 4x + \sin 6x)$

$= \sin x(1 + 2\cos 2x)\sin 4x$

$= \sin 4x(\sin x + 2\sin x\cos 2x)$

$= \sin 4x(\sin x + \sin 3x - \sin x)$

$= \sin 3x \sin 4x$

$x = \frac{\pi}{12}$ ආදේශ කිරීමෙන්

$$\sin\frac{\pi}{12}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1\right) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$\left(\frac{3+\sqrt{3}}{2}\right)\sin\frac{\pi}{12} = \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\sin\frac{\pi}{12} = \frac{2\sqrt{3}}{2\sqrt{2}(3+\sqrt{3})}$$

$$\sin\frac{\pi}{12} = \frac{\sqrt{3}(3-\sqrt{3})}{\sqrt{2}(9-3)}$$

$$= \frac{3(\sqrt{3}-1)}{6\sqrt{2}}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

සයින් සූත්‍රය :- ABC ත්‍රිකෝණයක් සඳහා සම්මත අංකනය අනුව

$$\frac{\text{Sin } A}{a} = \frac{\text{Sin } B}{b} = \frac{\text{Sin } C}{c} \quad \text{වේ}$$

$$\frac{\text{Sin } A}{a} = \frac{\text{Sin } B}{b} = \frac{\text{Sin } C}{c} = k$$

ආදේශ කිරීමෙන්

$a = b + \lambda c$ වූ

$\Rightarrow \text{Sin } A = \text{Sin } B + \lambda \text{Sin } C$

$\Rightarrow \lambda \text{Sin } C = \text{Sin } A - \text{Sin } B$

$= \text{Sin }(B + C) - \text{Sin } B$

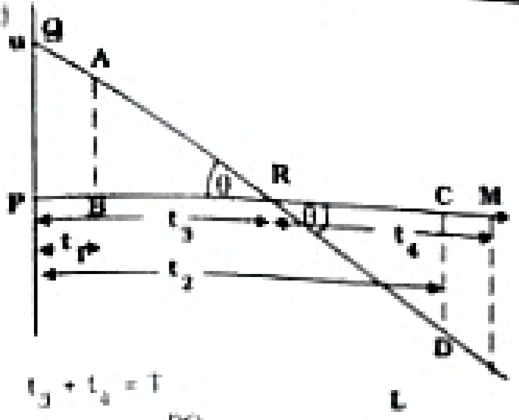
$(\because A + B + C = \pi)$

$$\Rightarrow 2\lambda \text{ Sin } \frac{C}{2}\cos\frac{C}{2} = 2\text{Sin } \frac{C}{2}\cos\frac{(2B+C)}{2}$$

$$\Rightarrow \lambda\cos\frac{C}{2} = \cos\left(B + \frac{C}{2}\right)$$

*** ***

(a) (i)



$t_3 + t_4 = T$

$\tan\theta = \frac{PQ}{PR} \Rightarrow g = \frac{u}{t_3}$

$t_3 = \frac{u}{g} \quad \therefore PQR\Delta = \frac{u^2}{2g}$

$\therefore RML\,\Delta = \frac{u^2}{2g}$ [illegible]

$\tan\theta = \frac{ML}{t_4} = g$

$RML\,\Delta = \frac{1}{2}\, t_4 \cdot g\, t_4 = \frac{u^2}{2g}$

$t_4^2 = \frac{u^2}{g^2} \Rightarrow t_4 = \frac{u}{g}$

$\therefore t_3 = t_4 = \frac{u}{g}$ [illegible] $T = \frac{2u}{g}$

[illegible]

$= PQR\,\Delta = \frac{u^2}{2g}$

$BR = \frac{u}{g} - t_1$

$RC = t_2 - \frac{u}{g} \quad \left(\because PR = \frac{u}{g} \right)$

t_1 හා t_2 [illegible]

$ABR\,\Delta = RCD\,\Delta$

$\therefore BR = RC$ [illegible]

$(\because \frac{1}{2}\; BR\, gBR = \frac{1}{2} RC.gRC,$ හා

$AB = g\,Br$ හා $CD = g\,RC$ [illegible]

$\frac{u}{g} - t_1 = t_2 - \frac{u}{g} \Rightarrow$

$t_1 + t_2 = \frac{2u}{g} = T$

(ii) $(A\,E) = \uparrow V$; $(B\,E) =$;

$(C\,E) =$; $(B\,A) =$;

$(C\,A) =$

$(B\,A) = (B\,E) + (E\,A)$

$= \qquad + \downarrow V$

$= \downarrow V \; + $

$= \overrightarrow{XY} + \overrightarrow{YZ_1}$

$= \overrightarrow{XZ_1}$

$(C\,A) = (C\,E) + (C\,A)$

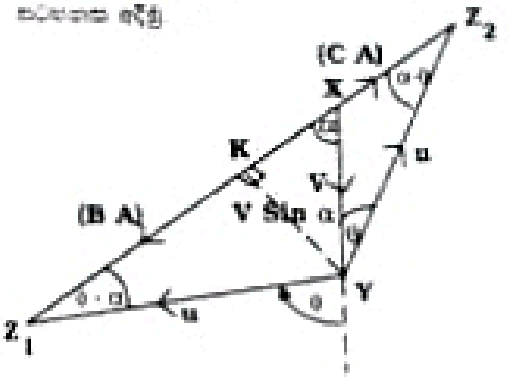
$= \qquad + \downarrow V$

$= \downarrow V \; + $

$= \overrightarrow{XY} + \overrightarrow{YZ_2}$

$= \overrightarrow{XZ_2}$

(b) XYZ_1 හා XYZ_2 [illegible]



(i) $XYZ_1\,\Delta$ [illegible]

$\frac{U}{\sin\alpha} = \frac{V}{\sin(\theta - \alpha)}$ — ①

[illegible] $XYZ_2\,\Delta$ [illegible]

$\frac{U}{\sin(\pi - \alpha)} = \frac{V}{\sin(\alpha - \theta)}$ — ②

① හා ② [illegible]

$\frac{U}{\sin\alpha} = \frac{V}{\sin(\theta - \alpha)} = \frac{V}{\sin(\alpha - \theta)}$

$(B\,C) = (B\,A) + (A\,C)$ වේ

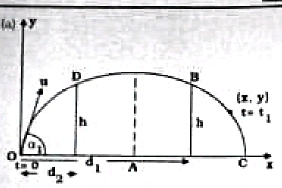
$= \overrightarrow{XZ_1} + (-\overrightarrow{XZ_2})$

$= \overrightarrow{XZ_1} + \overrightarrow{Z_2X} = \overrightarrow{Z_2Z_1}$

$YZ_1Z_2\,\Delta$ [illegible] $Z_1Z_2 = 2(Z_1K)$

$= 2\sqrt{u^2 - V^2\sin^2\alpha}$ $(KYZ_1\,\Delta$ [illegible]

01. (a)



O හරහා වූ x - y අක්ෂ [illegible]

$\rightarrow \quad x = u\cos\alpha_1 t_1$ ——— ①

$\uparrow \quad y = u\sin\alpha_1 t_1 - \frac{1}{2}gt_1^2$ ——— ②

① හා ② න් t_1 [illegible]

$y = x\tan\alpha_1 - \frac{gx^2}{2u^2}\sec^2\alpha_1$ ——— ③

③ [illegible]

$gx^2\sec^2\alpha_1 - 2u^2x\tan\alpha_1 + 2u^2y = 0$

[illegible] x හි [illegible]

$y = h$ විට [illegible]

$x = d_1$ හා $x = d_2$ වේ. $\because D = (d_2, h)$,

$B = (d_1, h)$

③ න් $x_1 + x_2 = \frac{2u^2\tan\alpha_1}{g\sec^2\alpha_1} = \frac{u^2}{g}\sin 2\alpha_1$

$= d_1 + d_2$ වේ. ——— ④

$x_1x_2 = \frac{2u^2h}{g\sec^2\alpha_1} = \frac{2hu^2}{g}\cos^2\alpha_1$

$= d_1d_2$ වේ.

$h \le \frac{u^2}{2g}\sin^2\alpha_1$ [illegible] d_1 හා d_2 [illegible]

[illegible] $x_1 = x_2 \Rightarrow d_1 = d_2$

එවිට ④ න් $2d_1 = \frac{u^2\sin 2\alpha_1}{g} \Rightarrow$

$d_1 = \frac{u^2\sin 2\alpha_1}{2g} = OA$

[illegible] α_2 විට Q [illegible] OA [illegible]

[illegible] OA $= \frac{u^2\sin 2\alpha_2}{2g}$ වේ.

එවිට $\frac{u^2\sin 2\alpha_1}{2g} = \frac{u^2\sin 2\alpha_2}{2g}$

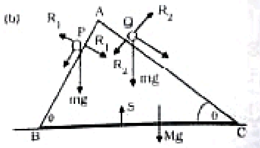
$\Rightarrow \sin 2\alpha_1 = \sin 2\alpha_2$

$\sin 2\alpha_1 = \sin(\pi - 2\alpha_2)$

$\Rightarrow 2\alpha_1 = \pi - 2\alpha_2 \Rightarrow \alpha_1 + \alpha_2 = \frac{\pi}{2}$

($\because \alpha_1 < \frac{\pi}{2}$ හා $\alpha_2 < \frac{\pi}{2}$ නිසා)

(b)



(P, ද) $= a_1$ (ϕ)

(Q, ද) $= a_2$ (θ)

(ද, E) $= \rightarrow f_1$

(P, E) $=$ (P, ද) $+$ (ද, E)

$= a_1$ (ϕ) $\rightarrow f_1$

(Q, E) $= a_2$ (θ) $+ \rightarrow f_1$

$\underline{F} = m\underline{a}$ [illegible]

P ⊙ $mg\sin\phi = m(a_1 - f_1\cos\phi)$ ——— ①

Q ⊙ $mg\sin\theta = m(a_2 + f_1\cos\theta)$ ——— ②

[illegible] $f_1 = 0$ [illegible]

① න් $a_1 = g\sin\phi$ ද ② න් $a_2 = g\sin\theta$ ද වේ.

[illegible] $\underline{F} = m\underline{a}$ [illegible]

$\rightarrow 0 = Mf_1 + m(f_1 + a_2\cos\theta) + m(f_1 - a_1\cos\phi)$ ——— ③

① න් $a_1 = g\sin\phi + f_1\cos\phi$ හා

② න් $a_2 = g\sin\theta - f_1\cos\theta$ නිසා

③ න් $0 = f_1(M + 2m) + m\cos\theta(g\sin\theta - f_1\cos\theta) - m\cos\phi(g\sin\phi + f_1\cos\phi)$

$mg(\sin\phi\cos\phi - \sin\theta\cos\theta) = f_1[M + 2m - m\cos^2\theta - m\cos^2\phi]$

$\frac{mg}{2}[\sin 2\phi - \sin 2\theta]$

$= f_1[M + m(1 - \cos^2\theta) + m(1 - \cos^2\phi)]$

$= f_1[M + m(\sin^2\theta + \sin^2\phi)]$

$f_1 = \frac{mg}{2}\frac{(\sin 2\phi - \sin 2\theta)}{[M + m(\sin^2\theta + \sin^2\phi)]}$

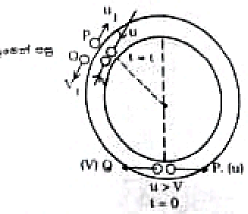
$\theta = \phi$ විට $2\theta = 2\phi$ [illegible]

$\sin 2\phi - \sin 2\theta = 0$

$\therefore f_1 = 0$ [illegible] P හා Q හි [illegible] ① හා ② න් $g\sin\phi$ හා $g\sin\theta$ [illegible]

02.

[illegible]



(a) P හා Q [illegible] ω_1 හා ω_2 [illegible]

$\omega_1 = \frac{u}{a}$ හා $\omega_2 = \frac{v}{a}$ වේ.

t [illegible]

$\omega_1 t + \omega_2 t = 2\pi$ [illegible] 2π [illegible]

[illegible]

$t\left(\frac{u}{a} + \frac{v}{a}\right) = 2\pi$

$\underline{t = \frac{2\pi a}{u + v}}$

[illegible]

[illegible]

$mu - mv = mv_1 - mu_1$

$\Rightarrow u - v = v_1 - u_1$

[illegible]

$e(u + v) = v_1 + u_1$

[illegible]

$v_1 = \frac{e(u + v) + u - v}{2} > 0$

[illegible] Q [illegible]

($\because u > v$ [illegible])

හා $u_1 = \frac{u(e - 1) + v(1 + e)}{2}$ වේ.

$u_1 > 0$ [illegible]

$u_1 = \frac{u(e - 1) + v(1 + e)}{2} > 0$ [illegible]

$e(u + v) > u - v \Rightarrow e > \frac{u - v}{u + v}$ [illegible]

[illegible] P හා Q [illegible]

$t = \frac{2\pi a}{u_1 + v_1}$ [illegible]

$e(u + v) = v_1 + u_1$ [illegible]

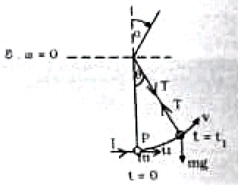
$t_1 = \frac{2\pi a}{e(u + v)}$

[illegible]

$= \frac{2\pi a}{u + v} + \frac{2\pi a}{e(u + v)}$

$= \frac{2\pi a}{(u + v)}\left(\frac{1 + e}{e}\right)$

(b)



P හි [illegible] u [illegible]

$\rightarrow \quad I = \Delta(mv)$ [illegible]

$I = mu - m(0) \Rightarrow u = \frac{I}{m}$ (P හි [illegible])

[illegible]

$t = 0$ හා $t = t_1$ [illegible]

$\frac{1}{2}mu^2 - mga = \frac{1}{2}mv^2 - mga\cos\theta$

$v^2 = u^2 - 2ga(1 - \cos\theta) = \frac{I^2}{m^2} - 2ga + 2ga\cos\theta$

$\underline{F} = m\underline{a}$ [illegible]

$T - mg\cos\theta = m\frac{v^2}{a}$

$T = \frac{m}{a}[u^2 - 2ga + 2ga\cos\theta] + mg\cos\theta$

$= \frac{m}{a}\left[\frac{I^2}{m^2} - 2ga + 3ga\cos\theta\right]$

$T = \frac{I^2}{ma} - 2mg + 3mg\cos\theta$ ——— ①

① න්

$$v^2 = \frac{I^2}{m^2} - 2ga + 2ga\cos\theta$$

T > 0 විය යුතුය.

එනම්, අවම T > 0 විය යුතුය

$$\frac{I^2}{ma} - 2mg - 3mg > 0 \quad (\because \text{අවම } \cos\theta = -1)$$

$$\underline{I^2 > 5m^2ga \Rightarrow I > m\sqrt{5ag}}$$

OP විට

$$\alpha = \pi - \theta \Rightarrow \theta = \pi - \alpha$$

විට $T = 0$ වේ. එවිට ① න්

$$0 = \frac{I^2}{ma} - 2mg + 3mg\cos(\pi - \alpha)$$

$$3mg\cos\alpha = \frac{I^2}{ma} - 2mg$$

$$\underline{\cos\alpha = \frac{I^2}{3m^2ag} - \frac{2}{3}} \text{ වේ.}$$

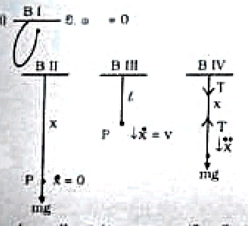
$0 < \alpha < \frac{\pi}{2}$ විට $0 < \cos\alpha < 1$ නිසා

$$0 < \frac{I^2}{3m^2ag} - \frac{2}{3} < 1 \Rightarrow \frac{2}{3} < \frac{I^2}{3m^2ag} < \frac{2}{3} + 1$$

$$\frac{2}{3} < \frac{I^2}{3m^2ga} < \frac{5}{3} \Rightarrow 2m^2ga < I^2 < 5m^2ga$$

$$\underline{m\sqrt{2ga} < I < m\sqrt{5ga}}$$

04. (i) B I දී, a = 0



I හා II අවස්ථා සඳහා ශක්ති සංස්ථිතිය යෙදීමෙන්,

$$0 + 0 + 0 = 0 - mgx + \frac{1}{2} \times 4mg\frac{(x-\ell)^2}{\ell}$$

$$mgx = \frac{4mg}{2}\,\frac{(x-\ell)^2}{\ell}$$

$$x\ell = 2(x^2 - 2x\ell + \ell^2)$$

$$0 = 2x^2 - 5x\ell + 2\ell^2$$

$$0 = (2x - \ell)(x - 2\ell)$$

$$x = \frac{\ell}{2} \text{ හෝ } x = 2\ell$$

$x > \ell$ නිසා උපරිම දිග $= 2\ell$

I හා III අවස්ථා සඳහා ශක්ති සංස්ථිතියෙන්

$$0 = \frac{1}{2}mv^2 - mg\ell$$

$$V^2 = 2g\ell$$

$$\underline{V = \sqrt{2g\ell}}$$

IV අවස්ථාව සලකමු.

$$T = \frac{4mg(x-\ell)}{\ell}$$

F = ma ↓ යෙදීමෙන්

$$mg - T = m\ddot{x}$$

$$mg - \frac{4mg}{\ell}(x - \ell) = m\ddot{x}$$

$$\frac{-4g}{\ell}\left(x - \frac{5\ell}{4}\right) = \ddot{x} \quad \text{———} ①$$

$x - \frac{5\ell}{4} = y$ නම් දෙවරක් අවකලනයෙන්

$$\ddot{x} = \ddot{y}$$

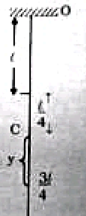
$$-\frac{4g}{\ell}y = \ddot{y}$$

① වලංගු වන්නේ $x \geq \ell$ විටය.

එනම් $y + \frac{5\ell}{4} \geq \ell$ විටය.

එනම් $y \geq \frac{-\ell}{4}$ විට

$$\ddot{y} + \frac{4g}{\ell}y = 0 \quad \text{———} ②$$



එම කේන්ද්‍රය $\ddot{y} = 0$ වන

එනම් y = 0 වන

O සිට $\frac{5\ell}{4}$ දුරින්

t = 0 විට $y = -\frac{\ell}{4}$

හා $\dot{y} = \sqrt{2g\ell}$ වේ.

$$y = A\cos\omega t + B\sin\omega t \quad \text{———} ③$$

අවකලනයෙන්

$$\dot{y} = -A\omega\sin\omega t + B\omega\cos\omega t \quad \text{———} ④$$

අවකලනයෙන්

$$\ddot{y} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$$

$$\ddot{y} = -\omega^2(A\cos\omega t + B\sin\omega t) \quad \text{———} ⑤$$

$$\ddot{y} = -\omega^2 y \quad \text{———} ⑥$$

② හා ⑥ න්

$$\underline{\omega^2 = \frac{4g}{\ell} \Rightarrow \omega = 2\sqrt{\frac{g}{\ell}}}$$

t = 0 විට $y = -\frac{\ell}{4}$ හා $\dot{y} = \sqrt{2g\ell}$ නිසා

③ න් $\quad \underline{-\frac{\ell}{4} = A(1) + B(0) \Rightarrow A = -\frac{\ell}{4}}$

④ න් $\quad \sqrt{2g\ell} = -A\omega(0) + B\omega(\cos 0)$

$$B = \frac{\sqrt{2g\ell}}{\omega} = \frac{\sqrt{2g\ell}}{2\sqrt{g/\ell}} = \frac{1}{2}\sqrt{2g\ell \times \frac{\ell}{g}}$$

$$= \frac{\ell\sqrt{2}}{2} = \frac{\ell}{\sqrt{2}}$$

එවිට

$$y = -\frac{\ell}{4}\cos\omega t + \frac{\ell}{\sqrt{2}}\sin\omega t$$

$$= \frac{3\ell}{4}\left(\frac{-\ell/4}{3\ell/4}\cos\omega t + \frac{\ell/\sqrt{2}}{3\ell/4}\sin\omega t\right)$$

$$= \frac{3\ell}{4}\left[-\sin\alpha\cos\omega t + \cos\alpha\sin\omega t\right]$$

$$= \frac{3\ell}{4}\sin(\omega t - \alpha)$$

මෙහි $\tan\alpha = \frac{1}{2\sqrt{2}}$

y උපරිම වන්නේ $\sin(\omega t - \alpha) = 1$ විටය.

$$\therefore \text{උපරිම } y = \frac{3\ell}{4}$$

තන්තුවේ උපරිම දිග $= \ell + \frac{\ell}{4} + \frac{3\ell}{4} = 2\ell$

$$\dot{y} = \frac{\ell\omega}{4}\sin\omega t + \frac{\ell}{\sqrt{2}}\omega\cos\omega t$$

$$= \frac{\sqrt{g\ell}}{2}\sin\omega t + \sqrt{2g\ell}\cos\omega t$$

$$= \frac{3}{2}\sqrt{g\ell}\,(-\sin\alpha\sin\omega t + \cos\alpha\cos\omega t)$$

$$\dot{y} = \frac{3}{2}\sqrt{g\ell}\cos(\omega t + \alpha)$$

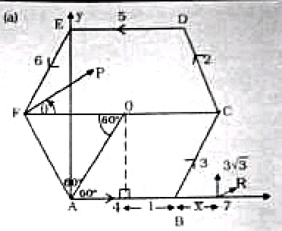
$\dot{y}$ උපරිම වන්නේ $\cos(\omega t + \alpha)$

උපරිම වන විටය.

උපරිම $\cos(\omega t + \alpha) = 1$ නිසා

$$\text{P හි උපරිම ප්‍රවේගය} = \frac{3\sqrt{g\ell}}{2}$$

05. (a)



R = 0 හා G ≠ 0 විට X හා Y යන දෙකම ශුන්‍ය විය යුතුයි

ඒ සඳහා → X = 4 + 3cos 60 - 2cos60 - 5 - 6cos60 + Pcosθ = 0

එනම් $P\cos\theta = \frac{7}{2}$ හා ↑Y

$$= 2\cos30 + 3\cos30 - 6\cos30 + P\sin\theta = 0$$

එනම්, $P\sin\theta = \frac{\sqrt{3}}{2}$ විය යුතුය.

$$P^2\cos^2\theta + P^2\sin^2\theta = \frac{49}{4} + \frac{3}{4}$$

$$= \frac{52}{4} = 13 \Rightarrow \underline{P = \sqrt{13}}$$

හා $\tan\theta = \frac{\sqrt{3}}{7}$ විය යුතුයි.

එවිට O වටා ඝූර්ණයේ විශාලත්වය G නම්,

$$G = (4 + 3 + 2 + 5 + 6)\,2\sin60 - P\sin\theta \times 2 + P\cos\theta \times 0$$

$$= 20 \times 2 \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times 2$$

$$= \underline{19\sqrt{3}}$$

P = 7 හා θ = 120° විට

AF දිගේ

$$\rightarrow X = 4 + \left(3 \times \frac{1}{2}\right) - \left(2 \times \frac{1}{2}\right) - 5 - \left(6 \times \frac{1}{2}\right) - \left(7 \times \frac{1}{2}\right)$$

$$= -1 + \frac{1}{2}(3 - 2 - 6 - 7)$$

$$X = -7 \text{ හා } \uparrow Y = 3\sqrt{3}$$

එවිට $R = \sqrt{X^2 + Y^2} = \sqrt{49 + 27}$

$= 2\sqrt{19} \neq 0$ නිසා

O වටා ඝූර්ණ ගැනීමෙන්,

$(4 + 3 + 2 + 5 + 6 - 7)\,2\sin 60$
$= 7 \times 2\sin 60 + 3\sqrt{3}\,(1 + \bar{x})$

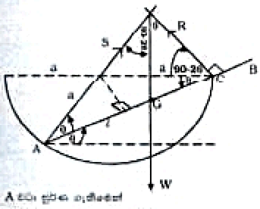
$\left(13 \times 2 \times \frac{\sqrt{3}}{2}\right) + \left(7 \times 3 \times \frac{\sqrt{3}}{3}\right)$

$= 3\sqrt{3} + 3\sqrt{3}\,\bar{x}$

$\bar{x} = \frac{17}{3}$

[illegible] AB [illegible] B සිට $5\frac{2}{3}$ m [illegible]

(b)



A වටා ඝූර්ණ ගැනීමෙන්

$R \times AC = W \times \ell\cos\theta$

(R [illegible] AB [illegible])

$R \times 2a\cos\theta = W \times \ell\cos\theta$

$R = \frac{W\ell}{2a}$ ——— ①

[illegible]

$\frac{W}{\sin(90 - \theta)} = \frac{R}{\sin(90 + 2\theta)} \Rightarrow$

$R = \frac{W\cos 2\theta}{\cos\theta}$ ——— ②

① හා ② න් $\frac{W\ell}{2a} = \frac{W\cos 2\theta}{\cos\theta} \Rightarrow$

$2a\cos 2\theta = \ell\cos\theta$

$2a(2\cos^2\theta - 1) = \ell\cos\theta \Rightarrow 4a\cos^2\theta - \ell\cos\theta - 2a = 0$

$\cos\theta = \frac{\ell \pm \sqrt{\ell^2 + 32a^2}}{8a}$

$0 < \theta < \pi/2$ නිසා $\cos\theta > 0$ [illegible]

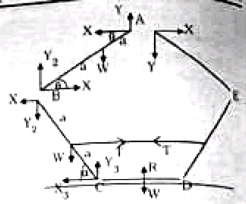
$\cos\theta = \frac{\ell + \sqrt{\ell^2 + 32a^2}}{8a}$

$CB = 2\ell - 2a\cos\theta$

$= 2\ell - 2a\left[\frac{\ell + \sqrt{\ell^2 + 32a^2}}{8a}\right]$

$= \frac{1}{4}\left[7\ell - \sqrt{\ell^2 + 32a^2}\right]$

06. (a)



$\theta = \frac{\pi}{5}$ හා $\alpha = \frac{2\pi}{5}$ වේ

[illegible] A [illegible]

$Y = 0$ ——— ①

BA [illegible]

B ↺ $W \times a\cos\theta = X \times 2a\sin\theta$

$X = \frac{W}{2}\cot\theta$

AB, BC [illegible]

C ↺ $(W \times a\cos\alpha) - (T \times a\sin\alpha) + X(2a\sin\alpha + 2a\sin\theta) - W(a\cos\theta - 2a\cos\alpha) = 0$

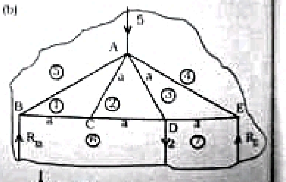
$W \times \cos\alpha - T\sin\alpha + \frac{W}{2}\cot\theta\,2\sin\alpha + \frac{W}{2}\cot\theta\,2\sin\theta - W\cos\theta + 2W\cos\alpha = 0$

$3W\cos\alpha + W\cos\theta + W\cot\theta\sin\alpha - W\cos\theta = T\sin\alpha$

$\frac{W(3\cos\alpha + \cot\theta\sin\alpha)}{\sin\alpha} = T$

$T = W(3\cot\alpha + \cot\theta)$
$= W(3\cot 2\pi/5 + \cot \pi/5)$

(b)



↑ $R_B + R_E = 7$

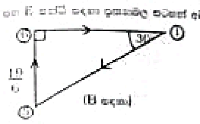
අ. පො. ස. (උසස් පෙළ) විභාගය - 2003 අගෝස්තු
සංයුක්ත ගණිතය II
ආදර්ශ පිළිතුරු

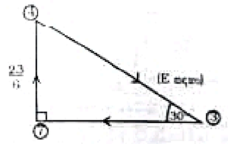
B ↺ $5 \times \frac{\sqrt{3}a}{2} + (2 \times 2a) = R_E \times 3a$

$R_E = \frac{23}{6}$ N

$\therefore R_B = \frac{19}{6}$ N

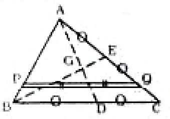
B හා E [illegible]





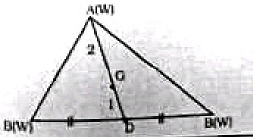
AB [illegible] 19/3 N
AE [illegible] 23/3 N

07.



ABC [illegible] BC [illegible] AD [illegible] AD [illegible] AC [illegible] BE [illegible]

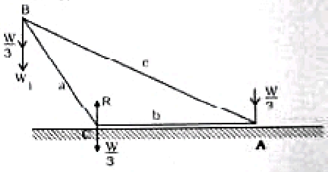
[illegible] G [illegible]



A, B හා C [illegible] $\frac{W}{g}$ (බර W) [illegible]

B හා C [illegible] W [illegible] 2W, D [illegible] (BD = DC) [illegible]

[illegible] A [illegible] W හා D [illegible] 2W [illegible] 3W, AG : GD = 2W : W = 2 : 1 [illegible] AD [illegible] G [illegible] A [illegible]



[illegible] W [illegible] A, B, C [illegible] $\frac{W}{3}$ [illegible]

[illegible] B [illegible] W_1 [illegible] CA [illegible] R [illegible] C [illegible]

C වටා ඝූර්ණ ගැනීමෙන්,

$\left(\frac{W}{3} + W_1\right) a\cos(180° - C) = \frac{W}{3} \times b$

$-a\left(\frac{W}{3} + W_1\right)\cos C = \frac{W}{3}b$

$-a\left(\frac{W}{3} + W_1\right)\left(\frac{a^2 + b^2 - c^2}{2ab}\right) = \frac{W}{3}b$

$\left(\frac{W}{3} + W_1\right)\left(\frac{c^2 - a^2 - b^2}{2b}\right) = \frac{W}{3}b$

$\frac{W}{3} + W_1 = \frac{2b^2W}{3(c^2 - a^2 - b^2)}$

$W_1 = \frac{2b^2W}{3(c^2 - a^2 - b^2)} - \frac{W}{3}$

$= \frac{W}{3}\frac{(2b^2 - c^2 + a^2 + b^2)}{(c^2 - a^2 - b^2)}$

$W_1 = \frac{W}{3}\frac{(a^2 + 3b^2 - c^2)}{(c^2 - a^2 - b^2)}$

08. (a) A හා B ස්වායත්ත සිද්ධීන් වන්නේ

$P(A \cap B) = P(A)\,P(B)$ නම් පමණි.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

X	0	1
P(X)	a	a

Y	0	1
P(Y)	b	b

X, 0 හා 1 අගයන් පමණක් ගන්නා බැවින්

$a + a = 1$ විය යුතුය

එනම් $a = \frac{1}{2}$ වේ. එලෙසම $b = \frac{1}{2}$ වේ

$A : X = 0$ හා $\bar{A} : X = 1$ බැවින්

A හා $\bar{A}$ [illegible]

[illegible]

$u = X + Y$ නම්

$X = 0$ හා $Y = 0$ විට $u = 0$

$X = 1$ හා $Y = 0$ විට $u = 1$

$X = 0$ හා $Y = 1$ විට $u = 1$

$X = 1$ හා $Y = 1$ විට $u = 2$

A හා B ස්වායත්ත බැවින් $\bar{A}$ හා $\bar{B}$ ද ස්වායත්ත වේ.

u	0	1	2
P(u = u)	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$	$\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$V = XY$ නම් $V = 0, 1$ අගයන් පමණක් ගනී

V	0	1
P(V = V)	$\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{4}$	$\left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{4}$

X	Y	
0	0	0
0	1	0
1	0	0
1	1	1

(b) $P(X/A) = 0.2$ හා $P(X/B) = 0.8$ බව දී ඇත.

$P(A) = \frac{40}{100} = 0.4$ හා $P(B) = \frac{60}{100} = 0.6$

බවද දී ඇත.

X [illegible] P(X)

$= P(X/A)\,P(A) + P(X/B)\,P(B)$

([illegible])

$= 0.2 \times 0.4 + 0.8 \times 0.6$

$= 0.08 + 0.48$

$= \underline{0.56}$

X [illegible] B [illegible]

$$P(B/X) = \frac{P(X/B)\,P(B)}{P(X/A)\,P(A) + P(X/B)\,P(B)}$$

([illegible])

$$\therefore P(B/X) = \frac{0.8 \times 0.6}{0.56} = \frac{0.48}{0.56}$$

$$= \frac{6}{7}$$

$P(B/X) > P(B)$ බැවින් X [illegible] B [illegible], B [illegible]

[illegible] $(p(A|X) = \frac{1}{7} < p(A) = 0.4$ බැවින් A [illegible])

09. [illegible] $x_1, x_2 \ldots\ldots x_n$ නම්

$$\text{මධ්‍යන්‍යය} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

[illegible]

[illegible]

$$S^2 = \frac{1}{N}\left\{\sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left[\sum_{i=1}^{N} x_i\right]^2\right\}$$

$\bar{x}$ [illegible]

$$\sum_{i=1}^{N} d_i = 0 \text{ වේ}$$

$$S^2 = \frac{1}{N}\left\{\sum_{i=1}^{N} (\bar{x} + d_i)^2 - \frac{1}{N}\left[\sum_{i=1}^{N} (\bar{x} + d_i)\right]^2\right\}$$

$$(\because d_i = x_i - \bar{x})$$

$$= \frac{1}{N}\left\{\sum_{i=1}^{N} (\bar{x}^2 + 2\bar{x}d_i + d_i^2) - \frac{1}{N}\left[\sum_{i=1}^{N} \bar{x} + \sum_{i=1}^{N} d_i\right]^2\right\}$$

$$= \frac{1}{N}\left\{\sum_{i=1}^{N} \bar{x}^2 + 2\bar{x}\sum_{i=1}^{N} d_i + \sum_{i=1}^{N} d_i^2 - \frac{1}{N}(N\bar{x})^2\right\}$$

$$= \frac{1}{N}\left\{N\bar{x}^2 + \sum_{i=1}^{N} d_i^2 - N\bar{x}^2\right\}$$

$$= \frac{1}{N}\sum_{i=1}^{N} d_i^2$$

$x_1 \le x_2 \le x_3 \le x_4 \le x_5$ යයි සිතමු.

[illegible] 31 බැවින්

$x_1 = 31 \ne x_2$ වේ

[illegible] 36 බැවින් $x_3 = 36$

[illegible]

$x_1 \ne x_2$ [illegible]

[illegible] x_4 හා x_5 වේ. $\therefore x_4 = x_5$

[illegible] 35 බැවින්

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 35$$

$$x_1 + x_2 + x_3 + x_4 + x_5 = 175$$

$$x_2 + x_4 + x_5 = 175 - 31 - 36$$

$$x_2 + 2x_4 = 108 \; (\because x_4 = x_5 \text{ නිසා})$$

$31 < x_2 < 36$ හා $x_4, x_5 > 36$ බැවින් [illegible]

[illegible]

$x_2 = 34$ හා $x_4 = x_5 = 37$

$x_2 = 32$ හා $x_4 = x_5 = 38$

[illegible] I 31, 32, 36, 38, 38 [illegible]

II 31, 34, 36, 37, 37 වේ.

$S^2 = 5.2$ බව දී ඇත.

I [illegible] $d_i = -4, -3, 1, 3, 3;$

$$S^2 = \frac{1}{5}(4 + 9 + 1 + 9 + 9)$$

$$= 6.4 \ne 5.2$$

II [illegible] $d_i = -4, -1, 1, 2, 2;$

$$S^2 = \frac{1}{5}(16 + 1 + 1 + 4 + 4)$$

$$= 5.2$$

[illegible] = 31, 34, 36, 37, 37

[illegible]

$$= \frac{3(\text{[illegible]} - \text{[illegible]})}{\text{[illegible]}}$$

$$= \frac{3(35 - 36)}{\sqrt{5.2}} = \frac{-3}{2.28} = -1.31$$

$$\bar{x} = \frac{1}{N}\sum x_i$$

$y_i = 55 - x_i$ බැවින්

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{N}\sum_{i=1}^{N}(55 - x_i)$$

$$\bar{y} = \frac{1}{N}\left(N \times 55 - \sum_{i=1}^{N} x_i\right)$$

$$\bar{y} = 55 - \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\bar{y} = 55 - \bar{x}$$

x_i	31	34	36	37	37
$Y_i = 55 - x_i$	24	21	19	18	18

$\bar{y} = 55 - 35 \; (\because \bar{x} = 35$ නිසා$)$

$= 20$

$d_i = y_i - \bar{y}$ නිසා d_i 4 1 -1 -2 2

[illegible] $d_i = x_i - \bar{x}$ [illegible] $-d_i$ [illegible] වේ.

[illegible] $S_y^2 = 5.2$ වේ (d_i^2 [illegible])

[illegible]

$$= \frac{3(20 - 19)}{\sqrt{5.2}}$$

$= 1.31$ වේ.

*** ***

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