01. (a) $px^2 + qx + r = 0$

[illegible] α, β [illegible]

$\alpha + \beta = \frac{-q}{p}, \ \alpha\beta = \frac{r}{p}$

$\alpha = \beta$ [illegible] $2\alpha = \frac{-q}{p}$ [illegible] $\alpha^2 = \frac{r}{p}$

එනම් $\alpha = \frac{-q}{2p}$ [illegible] $\alpha^2 = \frac{r}{p}$

[illegible] $\left(\frac{-q}{2p}\right)^2 = \frac{r}{p}$

$\frac{q^2}{4p^2} = \frac{r}{p}$

එනම් $q^2 = 4pr$

$\therefore px^2 + qx + r = 0$ [illegible]

$q^2 - 4pr = 0$ [illegible]

$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$

[illegible]

$b^2(c-a)^2 - 4\{a(b-c)\}\{c(a-b)\} = 0$

එනම් $b^2(c^2 - 2ac + a^2) - 4ac(ab - b^2 - ac + bc) = 0$

එනම් $b^2c^2 - 2ab^2c + a^2b^2 - 4a^2bc + 4ab^2c$
$+ 4a^2c^2 - 4abc^2 = 0$

එනම් $b^2c^2 + a^2b^2 + 4a^2c^2 + 2b^2ac - 4a^2bc - 4abc^2 = 0$
$(bc + ab - 2ac)^2 = 0$

$a^2b^2c^2$ [illegible] $\left(\frac{1}{a} + \frac{1}{c} - \frac{2}{b}\right)^2 = 0$

එනම් $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

(b) $E = a^3(b-c) + b^3(c-a) + c^3(a-b)$ [illegible]
b, c [illegible] E [illegible] a [illegible] $f(a)$ [illegible]
[illegible] $f(a) = a^3(b-c) + b^3(c-a) + c^3(a-b)$
[illegible] $f(b) = b^3(b-c) + b^3(c-b) + c^3(b-b) = 0$
[illegible] $f(c) = 0$

$f(-b-c) = -(b+c)^3(b-c) + b^3(c+b+c)$
$+ c^3(-b-c-b)$
$= -(b+c)^3(b-c) + (b+c)b^3$
$-(b+c)c^3 + b^3c - c^3b$
$= -(b+c)(b+c)^2(b-c) +$
$(b+c)(b^3 - c^3) + bc(b^2 - c^2)$
$= -(b+c)(b-c)[(b+c)^2 -$
$(b^2 + bc + c^2) - bc]$
$= -(b+c)(b-c)[b^2 + 2bc + c^2$
$- b^2 - bc - c^2 - bc]$
$= 0$

$\therefore (a-b), (a-c), (a+b+c)$ [illegible] $f(a)$ [illegible]

[illegible]
$f(a) = \lambda(a-b)(a-c)(a+b+c)$ [illegible]
a^3 [illegible] $\lambda = (b-c)$ [illegible]
$\therefore E = -(a-b)(b-c)(c-a)(a+b+c)$ [illegible]

02. (a) (i) [illegible] 12 [illegible] 7 [illegible]
[illegible]
[illegible]

[illegible] $= {}^{12}C_7$

$= \frac{12!}{7!\,5!}$

$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{7! \times 5 \times 4 \times 3 \times 2}$

$= 11 \times 9 \times 8$
$= \underline{792}$

(ii) [illegible] 12 [illegible] 6 [illegible]
[illegible] $= {}^{12}C_6$

[illegible]

$= \frac{1}{2} \times {}^{12}C_6$

$= \frac{1}{2} \times \frac{12!}{6!\,6!}$

$= \frac{1}{2} \times \frac{\cancel{12} \times 11 \times \cancel{10} \times \cancel{9} \times \cancel{8} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}}$

$= 11 \times 6 \times 7 = 11 \times 42 = \underline{462}$

(iii) [illegible]

[illegible] $= {}^{10}C_4$

$= \frac{10!}{6!\,4!}$

$= \frac{10 \times \cancel{9} \times \cancel{8} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{4} \times \cancel{3} \times \cancel{2}}$

$= \underline{210}$

(b) $(x+y)^n = \sum_{r=0}^{n} {}^nC_r \, x^{n-r} y^r$

[illegible] ${}^nC_r = \frac{n!}{r!\,(n-r)!}$

$r = 0, 1, 2, \ldots n$

$3^{2n+1} = 3(3^2)^n = 3(2+7)^n$

දැන් $x = 2,\ y = 7$ විට

$3(x+y)^n = 3(2+7)^n$

$= 3[2^n + {}^nc_1 2^{n-1} 7 + {}^nc_2 2^{n-2} 7^2 + \dots {}^nc_r 2^{n-r} 7^r + \dots {}^nc_n 7^n]$

$= 3(2^n) + 3.7 \sum_{r=1}^{n} {}^nc_r 2^{n-r} 7^{r-1}$

$= 3(2^n) + 7k$ ලෙස ගනිමු

මෙහි $k = 3 \sum_{r=1}^{n} {}^nc_r 2^{n-r} 7^{r-1} \in \mathbb{Z}$

දැන් $3^{2n+1} + 2^{n+2} = 7k + 3(2^n) + 2^{n+2}$

$= 7k + 2^n[3+4]$

$= 7k + 7.2^n$

$= 7(k + 2^n)$

<u>එමනිසා $3^{2n+1} + 2^{n+2}$, 7 න් බෙදේ</u>

03. $f(n) = p^{n+1} + (p+1)^{2n-1}$ යයි ගනිමු.

$n = 1$ විට $f(1) = p^2 + (p+1)$

$= p^2 + p + 1\ (\neq 0, \forall P \in \mathbb{Z})$

$\therefore n = 1$ විට $f(n)$, $p^2 + p + 1$ න් බෙදේ.

$n = k \in \mathbb{Z}^+$ විට ප්‍රතිඵලය සත්‍යයයි උපකල්පනය කරමු.

එවිට

$f(k) = p^{k+1} + (p+1)^{2k-1} = r(p^2+p+1)$ ලෙස ලිවිය හැකි මෙහි $r \in \mathbb{Z}$

දැන් $f(k+1) = p^{k+2} + (p+1)^{2k+1}$

$= p.p^{k+1} + (p+1)^2 (p+1)^{2k-1}$

$= p[p^{k+1} + p^{2k-1}] + (p^2+p+1)(p+1)^{2k-1}$

$= p[r(p^2+p+1)] + (p^2+p+1)(p+1)^{2k-1}$

$= (p^2+p+1)[rp + (p+1)^{2k-1}]$

$rp + (p+1)^{2k-1} \in \mathbb{Z}$ නිසා p^2+p+1 න් $f(k+1)$ බෙදේ

එනම් $n = k$ විට ප්‍රතිඵලය සත්‍ය නම් $n = k+1$ විට ද

ප්‍රතිඵලය සත්‍ය වේ

[illegible]

[illegible] n සඳහා $f(n) = p^{n+1} + (p+1)^{2n-1}$, p^2+p+1 න් බෙදේ.

(b) $U_r = \dfrac{r}{1+r^2+r^4}$

(i) $U_r = \dfrac{r}{1+r^2+r^4} = \dfrac{1}{2}\left[f(r) - \dfrac{1}{1+r+r^2}\right]$

$\therefore f(r) = \dfrac{2r}{1+r^2+r^4} + \dfrac{1}{1+r+r^2}$

$= \dfrac{2r}{(1+r+r^2)(1-r+r^2)} + \dfrac{1}{1+r+r^2}$

$= \dfrac{1}{1+r+r^2}\left(\dfrac{2r}{1-r+r^2} + 1\right)$

$= \dfrac{1}{1+r+r^2}\left[\dfrac{2r+1-r+r^2}{1-r+r^2}\right]$

$= \dfrac{1}{1-r+r^2}$

(ii) $f(r+1) = \dfrac{1}{1-(r+1)+(r+1)^2}$

$= \dfrac{1}{1+r+r^2}$

$\therefore U_r = \dfrac{1}{2}[f(r) - f(r+1)]$

(iii) $r = 1$ විට $U_1 = \dfrac{1}{2}[f(1) - f(2)]$

$r = 2$, $U_2 = \dfrac{1}{2}[f(2) - f(3)]$

$r = n-1$ $U_{n-1} = \dfrac{1}{2}[f(n-1) - f(n)]$

$r = n$ $U_n = \dfrac{1}{2}[f(n) - f(n+1)]$

$\sum_{r=1}^{n} U_r = \dfrac{1}{2}[f(1) - f(n+1)]$

$= \dfrac{1}{2}\left[1 - \dfrac{1}{1+n+n^2}\right]$

$= \dfrac{1}{2}\left[\dfrac{\cancel{1}+n+n^2-\cancel{1}}{1+n+n^2}\right]$

$= \dfrac{n(n+1)}{2(1+n+n^2)}$

04. $\dfrac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta}$

$= \dfrac{(\cos\alpha + i\sin\alpha)}{(\cos\beta + i\sin\beta)} \times \dfrac{(\cos\beta - i\sin\beta)}{(\cos\beta - i\sin\beta)}$

$= \dfrac{\cos\alpha\cos\beta + i\sin\alpha\cos\beta - i\cos\alpha\sin\beta + \sin\alpha\sin\beta}{\cos^2\beta + \sin^2\beta}$ $\quad (\because i^2 = -1)$

$= \dfrac{\cos\alpha\cos\beta + \sin\alpha\sin\beta + i(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{1}$

$= \cos(\alpha-\beta) + i\sin(\alpha-\beta)$

$\dfrac{Z_1}{Z_2} = \dfrac{-1+i}{1+i\sqrt{3}} \times \dfrac{1-i\sqrt{3}}{1-i\sqrt{3}} = \dfrac{-1+i\sqrt{3}+\sqrt{3}+i}{1+3}$

$= \dfrac{\sqrt{3}-1}{4} + i\dfrac{(\sqrt{3}+1)}{4}$

$Re\left(\dfrac{Z_1}{Z_2}\right) = \dfrac{\sqrt{3}-1}{4}$, —— ① $\quad Im\left(\dfrac{Z_1}{Z_2}\right) = \dfrac{(\sqrt{3}+1)}{4}$

$Z_1 = \sqrt{2}\left(\dfrac{-1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$

$Z_2 = 2\left(\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right) = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$

ඉහත සොයන් සාධනය අනුව

$\dfrac{Z_1}{Z_2} = \dfrac{\sqrt{2}}{2}\left\{\cos\left(\dfrac{3\pi}{4} - \dfrac{\pi}{3}\right) + i\sin\left(\dfrac{3\pi}{4} - \dfrac{\pi}{3}\right)\right\}$

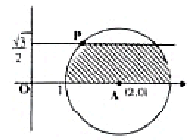
$= \dfrac{1}{\sqrt{2}}\left\{\cos\dfrac{5\pi}{12} + i\sin\dfrac{5\pi}{12}\right\}$

$Re\dfrac{Z_1}{Z_2} = \dfrac{1}{\sqrt{2}}\cos\dfrac{5\pi}{12}$ —— ②

① හා ② අනුව $Re\dfrac{Z_1}{Z_2} = \dfrac{1}{\sqrt{2}}\cos\dfrac{5\pi}{12} = \dfrac{\sqrt{3}-1}{4}$

$\therefore \cos\dfrac{5\pi}{12} = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

(b)



[illegible] P [illegible] Z_0 යයි ගනිමු. $|Z_0 - 2| = 1$ [illegible] A ලෙස ගනිමු.

එවිට, $Im\ Z_0 = \dfrac{\sqrt{3}}{2}$ හා $|Z_0 - 2| = 1$

එවිට, $Z_0 = X_0 + i\dfrac{\sqrt{3}}{2}$ $\quad 1 < X_0 < 2$ [illegible]

$|X_0 + i\dfrac{\sqrt{3}}{2} - 2| = 1$

$\Rightarrow (X_0 - 2)^2 + \dfrac{3}{4} = 1$

$\Rightarrow (X_0 - 2)^2 = \dfrac{1}{4}$

$\Rightarrow X_0 - 2 = \pm\dfrac{1}{2}$

$\Rightarrow X_0 = 2 \pm \dfrac{1}{2}$

$\Rightarrow X_0 = \dfrac{3}{2}$ $\quad (\because 1 < X_0 < 2$ නිසා$)$

එවිට $Z_0 = \dfrac{3}{2} + i\dfrac{\sqrt{3}}{2}$

දැන් $OP = |Z_0| = \sqrt{\dfrac{9}{4} + \dfrac{3}{4}} = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$

$OP^2 + PA^2 = 3 + 1 = 2^2 = OA^2$

$\therefore O\hat{P}A = \dfrac{\pi}{2}$

$\therefore OP$ [illegible] P හිදී [illegible]

$\therefore R$ [illegible] $Arg\ Z$ [illegible] P හිදී [illegible] $= Z_0$

$= \dfrac{3}{2} + i\dfrac{\sqrt{3}}{2}$

05 (a) (i) $y = (1+4x^2)\tan^{-1}2x$

$\dfrac{dy}{dx} = (1+4x^2) \times \dfrac{1}{1+4x^2} \times 2 + (\tan^{-1}2x) \times 8x$

$= 2 + 8x\tan^{-1}2x$

$= 2 + \dfrac{8xy}{(1+4x^2)}$

$\therefore (1+4x^2)\dfrac{dy}{dx} = 2(1+4x^2) + 8xy$

$(1+4x^2)\dfrac{dy}{dx} - 8xy = 2(1+4x^2)$ —— ①

(ii) ① න්

$(1+4x^2)\dfrac{d^2y}{dx^2} + 8x\dfrac{dy}{dx} - 8x\dfrac{dy}{dx} - 8y = 2(8x)$

$\Rightarrow (1+4x^2)\dfrac{d^2y}{dx^2} - 8y = 16x$ —— ②

(iii) ② න්

$(1+4x^2)\dfrac{d^3y}{dx^3} + 8x\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} = 16$

$x = 0$ විට $\left(\dfrac{d^3y}{dx^3}\right)_{x=0} - 8\left(\dfrac{dy}{dx}\right)_{x=0} = 16$

තවද ① න් $\left(\dfrac{dy}{dx}\right)_{x=0} = 2$

$\therefore \left(\dfrac{d^3y}{dx^3}\right)_{x=0} - 8 \times 2 = 16$

$\Rightarrow \left(\dfrac{d^3y}{dx^3}\right)_{x=0} = 32$

(b) [illegible] r cm හා [illegible] h cm [illegible]

$\pi r^2 h = 1024\pi$

$\therefore h = \dfrac{1024}{r^2}$

සිලින්ඩරයේ මුළු පෘෂ්ඨ වර්ගඵලය S නම්

$$S = 2\pi rh + 2\pi r^2$$
$$= 2\pi\left(r.\frac{1024}{r^2} + r^2\right) = 2\pi\left(\frac{1024}{r} + r^2\right)$$

එවිට $\frac{ds}{dr} = 2\pi\left(-\frac{1024}{r^2} + 2r\right)$

දැන් $\frac{ds}{dr} = 0 \Leftrightarrow \frac{1024}{r^2} = 2r \Leftrightarrow r^3 = 512 = 8^3$

$\Leftrightarrow r = 8$

දැන් $0 < r < 8$ විට $\frac{ds}{dr} < 0$

$8 < r$ විට $\frac{ds}{dr} > 0$

∴ මුළු පෘෂ්ඨ වර්ගඵලය අවමවන්නේ සිලින්ඩරයේ අරය 8 cm වේ.

06. (a) $I = \int_0^{\pi/2} \frac{dx}{3 + 2\cos x + \sin x}$ ලෙස ගනිමු

$t = \tan x/2$ ලෙස ගනිමු

$\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$ හා $dx = \frac{2\,dt}{1+t^2}$ වේ

එවිට $I = \int_0^1 \frac{\frac{2dt}{1+t^2}}{3 + 2\frac{(1-t^2)}{1+t^2} + \frac{2t}{1+t^2}} = \int_0^1 \frac{2dt}{3(1+t^2) + 2(1-t^2) + 2t}$

$$= 2\int_0^1 \frac{dt}{t^2 + 2t + 5} = 2\int_0^1 \frac{dt}{(t+1)^2 + 4}$$

$$= \left(\frac{2}{2}\tan^{-1}\left(\frac{t+1}{2}\right)\right)_0^1$$

$$= \underline{\frac{\pi}{4} - \tan^{-1}\frac{1}{2}}$$

(b) $I = \int e^{4x} \sin 3x\, dx$

කොටස් වශයෙන් අනුකලනයෙන්

$$I = \frac{1}{4}\int \sin 3x \frac{d}{dx}(e^{4x})\, dx$$
$$= \frac{1}{4}e^{4x}\sin 3x - \frac{1}{4}\int e^{4x}\, 3\cos 3x\, dx$$
$$= \frac{1}{4}e^{4x}\sin 3x - \frac{3}{4}\int \cos 3x \frac{d}{dx}\left(\frac{e^{4x}}{4}\right) dx$$
$$= \frac{1}{4}e^{4x}\sin 3x - \frac{3}{16}e^{4x}\cos 3x + \frac{3}{16}\int e^{4x}(-3\sin 3x)\, dx$$
$$= \frac{1}{4}e^{4x}\sin 3x - \frac{3}{16}e^{4x}\cos 3x - \frac{9}{16}\int e^{4x}\sin 3x\, dx$$
$$= \frac{e^{4x}}{16}[4\sin 3x - 3\cos 3x] - \frac{9}{16}I$$

$$\frac{25}{16}I = \frac{e^{4x}}{16}[4\sin 3x - 3\cos 3x]$$

$$I = \frac{e^{4x}}{25}[4\sin 3x - 3\cos 3x]$$

$\therefore \int e^{4x}\sin 3x\, dx = \frac{e^{4x}}{25}[4\sin 3x - 3\cos 3x] + C$

මෙහි C අභිමත නියතයකි

(c) $\frac{1}{x^3+1} = \frac{A}{(x+1)} + \frac{Bx + C}{x^2 - x + 1}$ ලෙස ගනිමු

එවිට $1 = A(x^2 - x + 1) + (Bx + C)(x + 1)$

එනම් $1 = (A + B)x^2 + (B - A + C)x + (A + C)$

x^2, x හා x^0 හි සංගුණක සැසඳීමෙන්

$A + B = 0$, $B - A + C = 0$, $A + C = 1$

සමීකරණ තුන විසඳීමෙන් $A = \frac{1}{3}$, $B = -\frac{1}{3}$, $C = \frac{2}{3}$

එවිට $\frac{1}{x^3+1} = \frac{1}{3}\left(\frac{1}{x+1} - \frac{(x-2)}{x^2 - x + 1}\right)$

$$= \frac{1}{3}\left(\frac{1}{x+1} - \frac{\frac{1}{2}(2x-1) - \frac{3}{2}}{x^2 - x + 1}\right)$$

$$= \frac{1}{3}\left(\frac{1}{x+1} - \frac{1}{2}\cdot\frac{2x-1}{x^2 - x + 1} + \frac{3}{2}\cdot\frac{1}{x^2 - x + 1}\right)$$

$$\therefore \int \frac{dx}{x^3+1} = \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{6}\int \frac{(2x-1)\,dx}{x^2 - x + 1} + \frac{1}{2}\int \frac{dx}{x^2 - x + 1}$$

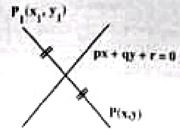
$$= \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{6}\int \frac{(2x-1)\,dx}{x^2 - x + 1} + \frac{1}{2}\int \frac{dx}{(x - \frac{1}{2})^2 + \frac{3}{4}}$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2 - x + 1) + \frac{1}{2}\frac{2}{\sqrt{3}}\tan^{-1}\frac{(x - \frac{1}{2})}{\frac{\sqrt{3}}{2}} + C$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2 - x + 1) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$$

C අභිමත නියතයකි.

07.



$px + qy + r = 0$ රේඛාව තුළින් $P_1(x_1, y_1)$ හි ප්‍රතිබිම්බය $P(x, y)$ යයි සිතමු. එවිට P_1P, $Px + qy + r = 0$ රේඛාවට ලම්බ වන අතර P_1P හි මධ්‍ය ලක්ෂ්‍යය එම රේඛාව මත වේ.

P_1 හරහා $Px + qy + r = 0$ ට ලම්බ ව ඇඳි රේඛාවේ සමීකරණය $y - y_1 = q/p\,(x - x_1)$

$\Rightarrow \frac{y - y_1}{q} = \frac{x - x_1}{p} = t$ (යයි සිතමු)

එවිට $x = x_1 + pt$, $y = y_1 + qt$ වේ

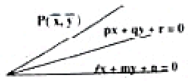
දැන් P_1P හි මධ්‍ය ලක්ෂ්‍යය $px + qy + r = 0$ මත බැවින්

$$p\left(x_1 + \frac{pt}{2}\right) + q\left(y_1 + \frac{qt}{2}\right) + r = 0$$

$$\Rightarrow px_1 + qy_1 + r + \frac{1}{2}(p^2 + q^2) = 0$$

$\Rightarrow t = -2\frac{(px_1 + qy_1 + r)}{p^2 + q^2} = -\lambda$ ලෙස ගනිමු

එවිට $\lambda = 2\frac{(px_1 + qy_1 + r)}{p^2 + q^2}$ හා $P \equiv (x_1 - \lambda p, y_1 - \lambda q)$ වේ



$px + qy + r = 0$ රේඛාව තුළින් $\ell x + my + n = 0$ රේඛාවේ ප්‍රතිබිම්බය මත පිහිටි ලක්ෂ්‍යයක් $P(\overline{x}, \overline{y})$ යයි සිතමු.

මෙවිට $px + qy + r = 0$ තුළින් P හි ප්‍රතිබිම්බය $\ell x + my + n = 0$ මත බැවින්

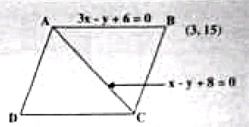
$$\ell(\overline{x} - \lambda p) + m(\overline{y} - \lambda q) + n = 0$$

මෙහි $\lambda = 2\frac{(p\overline{x} + q\overline{y} + r)}{p^2 + q^2}$ වේ.

එනම් $\ell\overline{x} + m\overline{y} + n - 2\frac{(p\overline{x} + q\overline{y} + r)(\ell p + mq)}{p^2 + q^2} = 0$

මෙලෙස $px + qy + r = 0$ රේඛාව තුළින් $\ell x + my + n = 0$ රේඛාවේ ප්‍රතිබිම්බය

$$\ell x + my + n - 2\frac{(\ell p + mq)(px + qy + r)}{p^2 + q^2} = 0$$



සමාන්තරාස්‍රයක විකර්ණ සමච්ඡේදනය වන බැවින් AD යනු $x - y + 8 = 0$ තුළින් $3x - y + 6 = 0$ හි ප්‍රතිබිම්බය වේ. එබැවින් AD හි සමීකරණය

$$3x - y + 6 - 2\frac{(1\times 3 + (-1)(-1))(x - y + 8)}{1 + 1} = 0$$

එනම් $3x - y + 6 - 4(x - y + 8) = 0$

$$x - 3y + 26 = 0$$

BC, AD ට සමාන්තර බැවින් BC හි සමීකරණය

$$y - 15 = \frac{1}{3}(x - 3)$$

එනම් $x - 3y + 42 = 0$

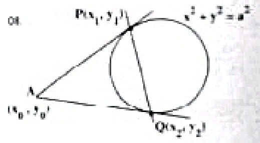
දැන් CD යනු $x - y + 8 = 0$ තුළින් $x - 3y + 42 = 0$ හි ප්‍රතිබිම්බය බැවින් එහි සමීකරණය

$$x - 3y + 42 - 2\frac{(1\times 1 + (-1)(-3))(x - y + 8)}{1 + 1} = 0$$

එනම් $x - 3y + 42 - 4(x - y + 8) = 0$

එනම් $\underline{3x - y - 10 = 0}$

08.



$x^2 + y^2 = a^2$ වෘත්තයට $A(x_0, y_0)$ සිට ඇඳි ස්පර්ශක $P(x_1, y_1)$ හා $Q(x_2, y_2)$ හි දී ස්පර්ශ වන්නේ යයි සිතමු.

(x_1, y_1) හා (x_2, y_2) හි දී ඇති වෘත්තයට ඇඳි ස්පර්ශක පිළිවෙලින් $xx_1 + yy_1 = a^2$ හා $xx_2 + yy_2 = a^2$ වේ.

ඒවා (x_0, y_0) ලක්ෂ්‍යය හරහා යන බැවින්

$x_0x_1 + y_0y_1 = a^2$ ——— ①

$x_0x_2 + y_0y_2 = a^2$ ——— ② වේ.

① හා ② අනුව $x_0x + y_0y = a^2$, (x_1, y_1) හා (x_2, y_2) ලක්ෂ්‍ය හරහා යන බව පෙනේ.

[illegible] ③ x හා y [illegible]

[illegible] ③ [illegible] (x_0, y_0) [illegible]

[illegible]

$\therefore (x_0, y_0)$ [illegible] $x^2 + y^2 = a^2$ [illegible]

[illegible]

$x_0 x + y_0 y = a^2$ වේ

$(1, 1)$ සහ $(-1, 0)$ [illegible]

$x^2 + y^2 + 2gx + 2fy + c = 0$ [illegible]

එවිට $1 + 1 + 2g + 2f + c = 0$ හා

$1 + 0 - 2g + 0 + c = 0$ වේ

ඒනම් $2g + 2f + c + 2 = 0$ —— ① හා

$2g - c - 1 = 0$ —— ② වේ

① හා ② න් $2g + 2f + 2g - 1 + 2 = 0$

ඒනම් $2f = -(4g + 1)$

එවිට [illegible]

$x^2 + y^2 + 2gx - (4g+1)y + 2g - 1 = 0$ —— ③

$S \equiv x^2 + y^2 - a^2 = 0$

දැන් $S = 0$ හා ③ [illegible]

$2gx - (4g+1)y + 2g - 1 + a^2 = 0$ —— ④

$R \equiv (x_0, y_0)$ [illegible] එවිට R [illegible] $S = 0$ [illegible]

[illegible]

$x_0 x + y_0 y - a^2 = 0$ [illegible] ④ [illegible]

[illegible] වේ.

එවිට $\dfrac{x_0}{2g} = \dfrac{y_0}{-(4g+1)} = \dfrac{-a^2}{2g - 1 + a^2}$

$x_0 = \dfrac{-2a^2 g}{2g - 1 + a^2}$ හා $y_0 = \dfrac{a^2(4g+1)}{2g - 1 + a^2}$

$2g\, x_0 + (a^2 - 1)x_0 = -2a^2 g \;,\; 2g y_0 + (a^2 - 1) y_0 = 4a^2 g + a^2$

$2g(x_0 + a^2) = (1 - a^2)x_0 \;;\; 2g(y_0 - 2a^2) = (1 - a^2)y_0 + a^2$

$\dfrac{(1 - a^2)x_0}{x_0 + a^2} = \dfrac{(1 - a^2)y_0 + a^2}{y_0 - 2a^2}$

$(1 - a^2)x_0(y_0 - 2a^2) = (x_0 + a^2)(1 - a^2)y_0 + (x_0 + a^2)a^2$

$\Rightarrow (1 - a^2)x_0 y_0 - 2(1 - a^2)a^2 x_0$

$= (1 - a^2)x_0 y_0 + (1 - a^2)a^2 y_0 + a^2(x_0 + a^2)$

$\Rightarrow -2a^2 x_0 + 2a^4 x_0 = a^2 y_0 - a^4 y_0 + a^2 x_0 + a^4$

$\Rightarrow (2a^2 - 3)x_0 + (a^2 - 1)y_0 - a^2 = 0$

$\therefore$ R [illegible] $(2a^2 - 3)x + (a^2 - 1)y - a^2 = \underline{0}$

[illegible] වේ.

09. (a) (i) $\sin 3\theta = \cos 2\theta$

$\Leftrightarrow \cos(\pi/2 - 3\theta) = \cos 2\theta$

[illegible] $\pi/2 - 3\theta = 2n\pi \pm 2\theta$ [illegible] $n \in \mathbb{Z}$

ඒනම් $5\theta = \pi/2(1 - 4n)$ හෝ $\theta = \pi/2(1 - 4n)$ —— ②

$\theta = \pi/10(1 - 4n)$ —— ①

$n = 0$ විට ① න් $\theta = \pi/10 = 18°$

$\therefore \theta = 18°$, $\sin 3\theta = \cos 2\theta$ [illegible]

[illegible] $\sin 3\theta = \cos 2\theta$

$3\sin\theta - 4\sin^3\theta = 1 - 2\sin^2\theta$

$4\sin^3\theta - 2\sin^2\theta - 3\sin\theta + 1 = 0$

$(\sin\theta - 1)(4\sin^2\theta + 2\sin\theta - 1) = 0$

[illegible] $\theta = 18°$ [illegible]

$0 < \sin 18° < 1$ [illegible] $\sin 18° - 1 \neq 0$

$\therefore \theta = 18°$, $4\sin^2\theta + 2\sin\theta - 1 = 0$ [illegible]

එවිට $\sin 18° = \dfrac{-2 \pm \sqrt{4 - 4 \cdot 4(-1)}}{2 \times 4}$

$= \dfrac{-2 \pm \sqrt{20}}{8}$

$= \dfrac{-1 \pm \sqrt{5}}{4}$

$\therefore \sin 18° = \underline{\underline{\dfrac{\sqrt{5} - 1}{4}}} \quad (\because 0 < \sin 18° < 1)$

(ii) $\tan^{-1}\dfrac{1}{3} = \alpha$ සහ $\tan^{-1}\left(\dfrac{1}{7}\right) = \beta$ [illegible]

එවිට, $\tan\alpha = \dfrac{1}{3}$, $\tan\beta = \dfrac{1}{7}$

$0 < \alpha < \dfrac{\pi}{4}$ හා $0 < \beta < \dfrac{\pi}{4}$ වේ

$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha} = \dfrac{2 \times \frac{1}{3}}{1 - \frac{1}{9}} = \dfrac{\frac{2}{3}}{\frac{8}{9}} = \dfrac{2}{3} \times \dfrac{9}{8} = \dfrac{3}{4}$

දැන් $2\tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{7} = 2\alpha + \beta$

එවිට $\tan(2\alpha + \beta) = \dfrac{\tan 2\alpha + \tan\beta}{1 - \tan 2\alpha \tan\beta}$

$= \dfrac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}} = \dfrac{\frac{25}{28}}{\frac{25}{28}} = 1$

$0 < 2\alpha + \beta < 3\pi/4$ [illegible] $2\alpha + \beta = \pi/4$

$\therefore 2\tan^{-1} 1/3 + \tan^{-1} 1/7 = \pi/4$ —— ①

$\tan^{-1} 2/11 = \delta$ නම් $\tan\delta = 2/11 \quad 0 < \delta < \pi/4$

එවිට $\tan^{-1} 1/7 + \tan^{-1} 2/11 = \beta + \delta$

දැන් $\tan(\beta + \delta) = \dfrac{\tan\beta + \tan\delta}{1 - \tan\beta\tan\delta} = \dfrac{\frac{1}{7} + \frac{2}{11}}{1 - \frac{1}{7}\cdot\frac{2}{11}} = \dfrac{\frac{25}{77}}{\frac{75}{77}}$

$= \dfrac{1}{3}$

$0 < \beta + \delta < \pi/2$ [illegible] $\beta + \delta = \tan^{-1}\dfrac{1}{3}$

$\tan^{-1}\dfrac{1}{7} + \tan^{-1}\dfrac{2}{11} = \tan^{-1}\dfrac{1}{3}$ —— ②

① හා ② $2\left(\tan^{-1}\dfrac{1}{7} + \tan^{-1}\dfrac{2}{11}\right) + \tan^{-1}\dfrac{1}{7} = \pi/4$

$3\tan^{-1}\dfrac{1}{7} + 2\tan^{-1}\dfrac{2}{11} = \pi/4$

(b) සයින් නීතිය -

ABC [illegible]

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ A, B, C [illegible]

[illegible] a, b, c [illegible]

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k$ [illegible]

එවිට $\sin^2 B + \sin^2 C - \sin^2 A = k^2(b^2 + c^2 - a^2)$

දැන් $L.H.S = \sin^2 B + (\sin C - \sin A)(\sin C + \sin A)$

$= \sin^2 B + 2\cos\dfrac{C + A}{2}\sin\dfrac{C - A}{2} \cdot 2\sin\dfrac{C + A}{2}\cos\dfrac{C - A}{2}$

$= \sin^2 B + \sin(C + A) \cdot \sin(C - A)$

$= \sin^2 B + \sin(\pi - B) \cdot \sin(C - A)$

$= \sin^2 B + \sin B \sin(C - A)$

$= \sin B\,[\sin B + \sin(C - A)]$

$= \sin B\,[2\sin\dfrac{B + C - A}{2}\cos\dfrac{B + A - C}{2}]$

$= 2\sin B \sin(\pi/2 - A)\cos(\pi/2 - C)$

$= 2\sin B \sin C \cos A = 2kb \cdot kc \cos A$

$= 2k^2 bc \cos A$

$\therefore 2k^2 bc \cos A = k^2(b^2 + c^2 - a^2)$

$\therefore \cos A = \underline{\underline{\dfrac{b^2 + c^2 - a^2}{2bc}}}$

(i) $\dfrac{b + c}{5} = \dfrac{c + a}{6} = \dfrac{a + b}{7} \Rightarrow$

$\dfrac{b + c}{5} = \dfrac{c + a}{6}$ $\qquad \dfrac{c + a}{6} = \dfrac{a + b}{7}$

$6b + 6c = 5c + 5a$ $\qquad 7c + 7a = 6a + 6b$

$c = 5a - 6b$ —— ① $\qquad 7c = -a + 6b$ —— ②

① + ② $8c = 4a$ $\qquad$ ② න් $6b = 7c + 2c$

$c = \dfrac{a}{2}$ $\qquad \dfrac{b}{3} = \dfrac{c}{2}$

$\therefore \dfrac{a}{4} = \dfrac{b}{3} = \dfrac{c}{2}$

$a = k\sin A$, $b = k\sin B$, $c = k\sin C$ [illegible]

$\dfrac{\sin A}{4} = \dfrac{\sin B}{3} = \dfrac{\sin C}{2}$

(ii) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{\frac{b}{c} + \frac{c}{b} - \frac{a}{b} \cdot \frac{a}{c}}{2}$

$= \dfrac{\frac{3}{2} + \frac{2}{3} - \frac{4}{3} \cdot \frac{4}{2}}{2}$

$= \dfrac{9 + 4 - 16}{6 \times 2} = \dfrac{-3}{12} = -\dfrac{1}{4}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac} = \dfrac{1}{2}\left(\dfrac{a}{c} + \dfrac{c}{a} - \dfrac{b}{a} \cdot \dfrac{b}{c}\right) = \dfrac{1}{2}\left(\dfrac{4}{2} + \dfrac{2}{4} - \dfrac{3}{4} \cdot \dfrac{3}{2}\right)$

$= \dfrac{1}{2}\left(\dfrac{16 + 4 - 9}{8}\right) = \dfrac{11}{16}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{1}{2}\left(\dfrac{a}{b} + \dfrac{b}{a} - \dfrac{c}{a} \cdot \dfrac{c}{b}\right)$

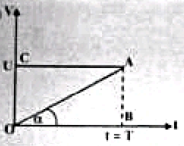
$= \dfrac{1}{2}\left(\dfrac{4}{3} + \dfrac{3}{4} - \dfrac{2}{4} \cdot \dfrac{2}{3}\right)$

$= \dfrac{1}{2} \times \dfrac{16 + 9 - 4}{12} = \dfrac{1}{2} \times \dfrac{21}{12} = \dfrac{7}{8}$

[illegible] $\underline{\dfrac{\cos A}{-1} = \dfrac{4\cos B}{11} = \dfrac{2\cos C}{7}}$

******

01. (a)

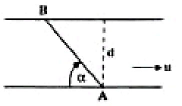


[illegible] $t = T$ [illegible]
[illegible] U [illegible].
$\therefore$ [illegible] $= \tan\alpha = \frac{U}{T}$

[illegible]
[illegible] $= OAC\Delta$

$= \frac{1}{2} UT$ m

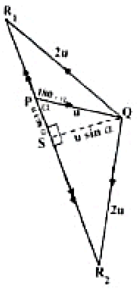
(b) (ජ E) $= \rightarrow u$
(B ජ) $= 2u$



A $\rightarrow$ B ට යාමට

(B E) $=$ ↖α
(B E) $=$ (B ජ) $+$ (ජ E)

↖α $= 2u + \rightarrow u$
$= \rightarrow u + 2u$
$= \overrightarrow{PQ} + \overrightarrow{QR_1}$
$= \overrightarrow{PR_1}$



B $\rightarrow$ A ට පැමිණීමට

(B E) $=$ ↘α
↘α $= 2u + \rightarrow u$
$= \rightarrow u + 2u$
$= \overrightarrow{PQ} + \overrightarrow{QR_2}$
$= \overrightarrow{PR_2}$

(i) A සිට B ට යාමට ගතවන කාලය, B සිට A ට පැමිණීමට ගතවන කාලය මෙන් දෙගුණයක් නිසා

[illegible] $|\overrightarrow{PR_2}| = 2|\overrightarrow{PR_1}|$ විය යුතුයි. —— ①

$\left(\because \frac{AB}{|\overrightarrow{PR_1}|} = 2\frac{AB}{|\overrightarrow{PR_2}|} \text{ [illegible]} \right)$

$|\overrightarrow{PR_1}| = \sqrt{4u^2 - u^2\sin^2\alpha} - u\cos\alpha$ —— ②

$|\overrightarrow{PR_2}| = \sqrt{4u^2 - u^2\sin^2\alpha} + u\cos\alpha$

① ට අනුව

$\sqrt{4u^2 - u^2\sin^2\alpha} + u\cos\alpha = 2\left[\sqrt{4u^2 - u^2\sin^2\alpha} - u\cos\alpha\right]$

$3u\cos\alpha = \sqrt{4u^2 - u^2\sin^2\alpha}$

$9u^2\cos^2\alpha = u^2(4 - \sin^2\alpha)$

$9(1 - \sin^2\alpha) = 4 - \sin^2\alpha$

$5 = 8\sin^2\alpha$

$\therefore \sin\alpha = \sqrt{\frac{5}{8}}$ ($\alpha < \pi/2$ නිසා)

(ii) $|\overrightarrow{PR_1}| = \sqrt{4u^2 - u^2\frac{5}{8}} - u\sqrt{\frac{3}{8}}$ (② න්)

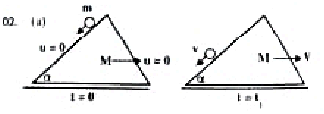
$\sqrt{\frac{27u^2}{8}} - u\sqrt{\frac{3}{8}} = \frac{3\sqrt{3}}{\sqrt{8}}u - \frac{\sqrt{3}u}{\sqrt{8}} = \frac{2\sqrt{3}}{8} = \sqrt{\frac{3}{2}}u$

A $\rightarrow$ B ට කාලය 2t නම් ගතවන මුළු කාලය 3t වේ.

$2t = \frac{d\,\text{cosec}\,\alpha}{|\overrightarrow{PR_1}|} \Rightarrow 2t = d\sqrt{\frac{8}{5}} \Big/ \sqrt{\frac{3}{2}}u$ (II [illegible])

$\therefore 3t = \frac{3}{2}\sqrt{\frac{8}{5}} \times \sqrt{\frac{2}{3}}\frac{d}{u} = \frac{6d}{u\sqrt{15}}$

02. (a)



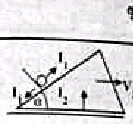
(ද E) $= \rightarrow V$ (අ. ද) $=$ ↙α u
(අ. E) $=$ ↙α u $+ \rightarrow V =$ ↑$u\sin\alpha$, $\rightarrow V - u\cos\alpha$

[illegible] ($t = 0$ හා $t = t_1$) [illegible]

$\rightarrow \quad 0 = MV + m(V - u\cos\alpha)$

$\frac{m u\cos\alpha}{M + m} = V$

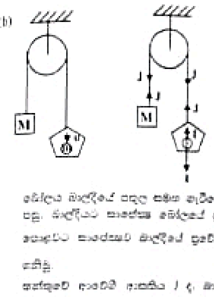
[illegible] V_1 නම්



$0 = (M + m)V_1 \Rightarrow V_1 = 0$ [illegible]
[illegible] I_2 නම්
$I = \Delta(mv)$ [illegible] $\uparrow I_2 = (M + m)0 - [M \times 0 + m(-u\sin\alpha)]$

$I_2 = mu\sin\alpha$

(b)



[illegible] $\uparrow V_1$ ද [illegible] $\downarrow V_2$ ද යයි ගනිමු.
[illegible] J ද, [illegible] I ද යයි ගනිමු.

$\therefore$ (අංශු E) $= \uparrow V_1 = eu$ වේ.
(අංශු E) $=$ (අංශු [illegible]) $+$ ([illegible] E)
$= \uparrow eu + \downarrow V_2 = \uparrow(eu - V_2)$
$I = \Delta(mV)$

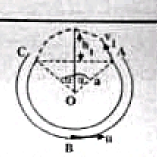
[illegible] $\uparrow I = m(eu - V_2) - m(-u)$
$I = mu(1 + e) - mV_2$ —— ①
[illegible] $\uparrow J - I = M(-V_2)$ —— ②
[illegible] $\uparrow J = MV_2$ —— ③
① + ② $J = mu(1 + e) - V_2(M + m)$ —— ④
③ = ④ $MV_2 = mu(1 + e) - V_2(M + m)$

$\therefore V_2 = \frac{mu(1 + e)}{2M + m}$

[illegible] $J = \frac{M\,mu(1 + e)}{2M + m}$; ③ න්
[illegible] 0 ද වේ. $s = ut + \frac{1}{2}at^2$ [illegible] $s = 0, u = eu, a = -g, t = t_1$ නම්

$0 = eut_1 - \frac{1}{2}gt_1^2 \Rightarrow t_1 = \frac{2eu}{g}$

03.



[illegible] A [illegible] V_1 යයි ගනිමු. [illegible] ↖α වේ.
$AC = 2a\sin\alpha$
B සිට A [illegible]

$\frac{1}{2}mu^2 - mga = \frac{1}{2}mV_1^2 + mga\cos\alpha$

$V_1^2 = u^2 - 2ga(1 + \cos\alpha)$ —— ①

A $\rightarrow$ C [illegible]
$S = ut + \frac{1}{2}at^2$ [illegible]
↑ $S = 0, u = V_1\sin\alpha, a = -g, t = t_1$
$0 = V_1\sin\alpha\, t_1 - \frac{1}{2}gt_1^2$

$t_1 \neq 0$ නිසා $t_1 = \frac{2V_1\sin\alpha}{g}$ —— ②

$\rightarrow$ $S = 2a\sin\alpha, U = V_1\cos\alpha, t = t_1, a = 0$

$2a\sin\alpha = V_1\cos\alpha\frac{2V_1\sin\alpha}{g}$; ② න්

$\frac{ag}{\cos\alpha} = V_1^2$ —— ③

① හා ③ න් $\frac{ag}{\cos\alpha} = u^2 - 2ga(1 + \cos\alpha)$
$ag\sec\alpha + 2ga(1 + \cos\alpha) = u^2$
$ga[2(1 + \cos\alpha) + \sec\alpha] = u^2$

A සිට [illegible] $v^2 = u^2 + 2as$ [illegible]
$v = 0, u_2 = V_1\sin\alpha, a = -g, s = h_1$ නම්

$0 = V_1^2\sin^2\alpha - 2gh_1 \Rightarrow h_1 = \frac{V_1^2\sin^2\alpha}{2g}$

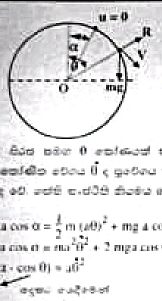
0 සිට උස
$= h_1 + a\cos\alpha$

$= \frac{ag\sin^2\alpha}{\cos\alpha \times 2g} + a\cos\alpha = \frac{ag(\sin^2\alpha + 2\cos^2\alpha)}{2g\cos\alpha}$

$= \frac{a(\sin^2\alpha + \cos^2\alpha + \cos^2\alpha)}{2\cos\alpha}$

$= \frac{a}{2}\frac{(1 + \cos^2\alpha)}{\cos\alpha} = \frac{a}{2}(\sec\alpha + \cos\alpha)$

(ii)



OP ලම්බ සිරස සමග θ කෝණයක් සාදන විට අංශුවේ කෝණික වේගය $\dot{\theta}$ ද ප්‍රවේගය v නම් $v = a\dot{\theta}$ ද වේ. ශක්ති සංස්ථිති නියමය යෙදීමෙන්

$$0 + mg\,a\cos\alpha = \frac{1}{2}m(a\dot{\theta})^2 + mg\,a\cos\theta$$

$$2mg\,a\cos\alpha = ma^2\dot{\theta}^2 + 2mga\cos\theta$$

$$2g(\cos\alpha - \cos\theta) = a\dot{\theta}^2$$

$F = ma$ යෙදීමෙන්

$$mg\cos\theta - R = ma\dot{\theta}^2$$

$$R = mg\cos\theta - m.2g(\cos\alpha - \cos\theta)$$

$$R = mg(3\cos\theta - 2\cos\alpha)$$

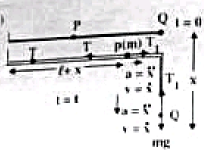
අංශුව ගෝලය අතහැර යයි නම් $R = 0$ වේ

එවිට $\theta = \theta_1$ නම්

$$\cos\theta_1 = \frac{2}{3}\cos\alpha$$

$$\underline{\theta_1 = \cos^{-1}\left(\frac{2}{3}\cos\alpha\right)}$$ වේ

(b)



P හා Q වේග $\dot{x}$ ද, ත්වරණය $\ddot{x}$ ලෙසින් ද වේ.

ශක්ති සංස්ථිති මූලධර්මය යෙදීමෙන්

$$0 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{x}^2 - mgx + \frac{1}{2}mg\frac{x^2}{\ell}$$

$$2\cancel{m}\dot{x}^2 = 2\cancel{m}gx - \cancel{m}g\frac{x^2}{\ell}$$

$$2\dot{x}^2 = \frac{g}{\ell}[2\ell x - x^2]$$

$$\dot{x}^2 = \frac{g}{2\ell}[\ell^2 - (\ell - x)^2]$$

$$\dot{x}^2 = \omega^2[\ell^2 - (\ell - x)^2] \text{ —— ①}$$

t විෂයෙන් අවකලනයෙන්

$$2\dot{x}\,\ddot{x} = \omega^2(2)(\ell - x)(-\dot{x})$$

$$\ddot{x} = \omega^2(\ell - x)$$

$$\ddot{x} = -\omega^2(x - \ell)$$

මෙය සරල අනුවර්තී චලිතයක සමීකරණය යයි. ∴ චලිතය සරල අනුවර්තී වේ. කේන්ද්‍රය $\ddot{x} = 0$ වන 0 සිට $x = 2\ell$ දුරකදී ය විස්තාරය ① ට අනුව $\dot{x} = 0$ වන විට

① න් $x^2 - 2x\ell = 0$ එනම් $x = 2\ell$ විට

මෙය මත ඇත්තේ 0 සිට මැඩින් විස්තාරය ℓ වේ. අනුවර්ත චලිතය සැලකීමෙන් P අංශුව මේස දාරයට ළඟාවීමට ගත

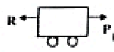
$$\text{කරන කාලය } t_1 = \frac{\frac{\pi}{2}}{\omega} = \frac{\frac{\pi}{2}}{\sqrt{\frac{g}{2\ell}}} = \pi\sqrt{\frac{\ell}{2g}}$$

මේස දාරය පසු ස. අ. ච. කේන්ද්‍රය මැඩින්

$$v^2 = \omega^2(\ell^2 - y^2) \text{ න් } y = 0 \text{ විට } v^2 = \frac{g}{2\ell}(\ell^2 - 0) = \frac{g}{2}\ell$$

$$\therefore \underline{v = \sqrt{\frac{g\ell}{2}}}$$

04. (a) $160\ \text{km h}^{-1} = \frac{400}{9}\ \text{ms}^{-1}$



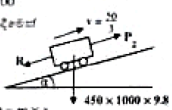
$$P_1 = \frac{3000 \times 1000}{400/9}\text{N}$$

$\underline{F} = m\underline{a}$ යෙදීමෙන්

$$P_1 - R = m \times 0$$

$$3000 \times 1000 \times \frac{9}{400} = R \Rightarrow R = 67500\ \text{N}$$

$\underline{F} = m\underline{a}$ යෙදීමෙන්



$$P_2 - R - mg\sin\alpha = m \times a_1$$

$$P_2 = \frac{3000 \times 1000}{50/3}\ \text{N}$$

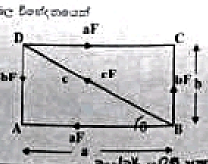
$$\cancel{3000} \times 10\cancel{00} \times \frac{3}{5\cancel{0}} - 675\cancel{00} - 450 \times 10\cancel{00} \times 9.8 \times \frac{1}{70} = 450 \times 10\cancel{00} \times a_1$$

$$1800 - 675 - 450\cancel{0} \times \frac{98}{700} = 4500\,a_1$$

$$495 = 4500\,a_1$$

$$\underline{a_1 = 0.11\ \text{ms}^{-2}}$$

05. (a) බල විභේදනයෙන්



$$\vec{X} = aF - aF - cF\cos\theta$$

$$= -cF\frac{a}{c}$$

$$= -aF$$

$$\uparrow Y = bF - bF + cF\sin\theta$$

$$= cF\frac{b}{c}$$

$$= bF$$

සම්ප්‍රයුක්තය $R = \sqrt{X^2 + Y^2}$

$$= F\sqrt{a^2 + b^2}$$

$$= Fc$$

සම්ප්‍රයුක්ත බලය AB සමඟ α කෝණයක් සාදයි නම්,

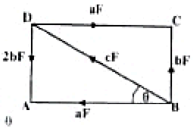
$$\tan\alpha = \frac{bF}{-aF} = \frac{-b}{a} = -\tan\theta = \tan(180 - \theta)$$

∴ BD දිශාවට වේ

B වටා ඝූර්ණයන්ගේ ඓක්‍යය $= aF \times b - bF \times a = 0$

∴ සම්ප්‍රයුක්තය $\overrightarrow{BD}$ දිගේ පිහිටයි.

නව බල පද්ධතිය විභේදනයෙන්



$X = -aF$ නොවෙනස්ව

$$\uparrow Y = -2bF + bF + cF\sin\theta$$

$$= -bF + cF\frac{b}{c}$$

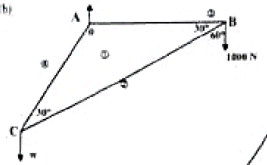
$$= 0$$

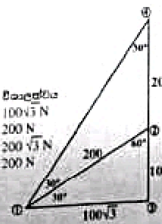
∴ සම්ප්‍රයුක්තය B සිට A දෙසට aF වේ

D වටා ඝූර්ණයන්ගේ ඓක්‍යය $= bF \times a - aF \times b = 0$

∴ නව සම්ප්‍රයුක්තය CD දිගේ aF වේ.

(b)





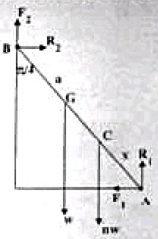
දණ්ඩ	ප්‍රත්‍යාබලය	විශාලත්වය
AB	ආතතිය	$100\sqrt{3}$ N
BC	තෙරපුම	200 N
AC	ආතතිය	$200\sqrt{3}$ N
W		200 N

06. බිත්තියට A සිට x දුරක් ඇති C හි මිනිසෙක් සිටි විට A හා B ලක්ෂ්‍ය දෙකෙහිදීම සීමාකාරී සමතුලිතතාවයේ ඇත.

$$\therefore F_1 = \mu R_1 \text{ —— ①}$$

$$F_2 = \mu R_2 \text{ —— ②}$$

ඉණිමගේ සමතුලිතතාවය සලකා



$$\overset{\curvearrowleft}{A}\quad nWx\frac{1}{\sqrt{2}} + W.a\frac{1}{\sqrt{2}} - F_2\,2a\frac{1}{\sqrt{2}} - R_2\,2a\frac{1}{\sqrt{2}} = 0 \text{ —— ③}$$

② න් $\frac{W}{\sqrt{2}}(nx + a) = 2a\frac{R_2}{\sqrt{2}}(\mu + 1)$

$$\therefore R_2 = \frac{W(nx + a)}{2a(\mu + 1)} \text{ —— ④}$$

$\rightarrow\quad F_1 = R_2$ —— ⑤

$\uparrow\quad F_2 + R_1 = W + nW$ —— ⑥

⑥ න් $R_1 = W(1 + n) - F_2$ —— ⑦

② න් $F_2 = \frac{\mu W(nx + a)}{2a(\mu + 1)}$

⑦ න් $R_1 = W(1 + n) - \frac{\mu W(nx + a)}{2a(\mu + 1)}$

$$R_1 = \frac{W[2a(1+n)(1+\mu) - \mu(nx + a)]}{2a(\mu + 1)}$$

① න් $\frac{W(nx + a)}{2a(\mu + 1)} = \mu W\frac{[2a(1+n)(1+\mu) - \mu(nx + a)]}{2a(\mu + 1)}$

$$nx + a = \mu 2a(1+n)(1+\mu) - \mu^2(nx + a)$$

$$nx + \mu^2 nx = \mu 2a[1 + \mu + n + n\mu] - \mu^2 a - a$$

$$= 2\mu a + 2\mu^2 a + 2\mu na + 2\mu^2 na - \mu^2 a - a$$

$$nx(1 + \mu^2) = a[2\mu^2 n + \mu^2 + 2\mu n + 2\mu - 1]$$

$$= a[\mu^2(2n + 1) + 2\mu(n + 1) - 1]$$

$$\therefore x = \frac{a}{n(1 + \mu^2)}[\mu^2(2n + 1) + 2\mu(n + 1) - 1]$$

$\mu = \frac{1}{2}$ විට

$$x = \frac{a}{n(1 + \frac{1}{4})}\left[\frac{1}{4}(1 + 2n) + \cancel{1} + n - \cancel{1}\right]$$

$$= \frac{4a}{5n}\left(\frac{1}{4} + \frac{6n}{4}\right)$$

$$= \frac{a}{5n}(1 + 6n)$$

[illegible] $x > 2a$ [illegible]

[illegible] $\frac{a}{5a}(1 + 6a) > 2a$ [illegible]

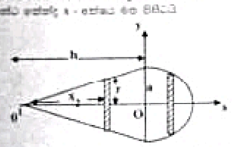
$1 + 6a > 10a$

$1 > 4a$

$a < \frac{1}{4}$ [illegible]

$a = \frac{1}{4}$ [illegible]

07. [illegible]
[illegible] x - [illegible]



(i) [illegible] y [illegible] x_1 [illegible]
[illegible] δx_1 [illegible]
[illegible] $= \pi(a^2 - x_1^2)\,\delta x_1$
[illegible] ρ [illegible]
[illegible] $= \pi(a^2 - x_1^2)\,\delta x_1\,\rho$
[illegible]
y [illegible] $= x_1$
[illegible] $(\bar{x}, 0)$ [illegible]

$$\bar{x} = \frac{\int_0^a \pi(a^2 - x_1^2)\,dx_1\,\rho \times x_1}{\int_0^a \pi(a^2 - x_1^2)\,dx_1\,\rho}$$

$$= \frac{\rho\pi\left(\frac{a^2x_1^2}{2}\right)_0^a - \rho\pi\left(\frac{x_1^4}{4}\right)_0^a}{\rho\pi[a^2x_1]_0^a - \rho\pi\left(\frac{x_1^3}{3}\right)_0^a}$$

$$= \frac{\frac{a^4}{2} - \frac{a^4}{4}}{a^3 - \frac{a^3}{3}} = \frac{3a}{8}$$

$\therefore$ [illegible] $= \left(\frac{3a}{8}, 0\right)$

(ii) [illegible] 0^1 [illegible] $\bar{\bar{X}}$ [illegible] 0^1 [illegible] x_2 [illegible] δx_2
[illegible] r [illegible]
[illegible] Δ [illegible] $\frac{x_2}{r} = \frac{h}{a} \Rightarrow r = \frac{ax_2}{h}$

[illegible] $= \pi r^2 \delta x_2 \rho$

$$= \pi\frac{a^2x_2^2}{h^2}\,\delta x_2\rho$$

[illegible] 0^1 [illegible] $= x_2$

$$\bar{\bar{x}} = \frac{\int_0^h \frac{\pi a^2 x_2^2}{h^2}\,dx_2\,\rho \times x_2}{\int_0^h \frac{\pi a^2 x_2^2}{h^2}\,dx_2\,\rho}$$

$$\bar{\bar{x}} = \frac{\frac{\pi a^2}{h^2}\int_0^h x_2^3\,dx_2}{\frac{\pi a^2}{h^2}\int_0^h x_2^2\,dx_2} = \frac{\left(\frac{x_2^4}{4}\right)_0^h}{\left(\frac{x_2^3}{3}\right)_0^h} = \frac{3h}{4}$$

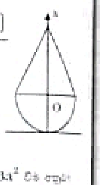
$\therefore$ O [illegible] $= h - \frac{3h}{4} = \frac{h}{4}$

[illegible] $= \left(-\frac{h}{4}, 0\right)$ [illegible]

[illegible] O [illegible] x_3 [illegible]

[illegible]

$$x_3 = \frac{\frac{1}{3}\pi a^2 h\rho\,\frac{h}{4} + \frac{2}{3}\pi a^3\rho\left(-\frac{3}{8}a\right)}{\frac{1}{3}\pi a^2 h\rho + \frac{2}{3}\pi a^3\rho}$$



$$= \frac{\frac{1}{3}\pi a^2\rho\left(\frac{h^2}{4} - \frac{3}{4}a^2\right)}{\frac{1}{3}\pi a^2\rho\,(h + 2a)} = \frac{h^2 - 3a^2}{4(h + 2a)}$$

$\therefore x_3 = \frac{h^2 - 3a^2}{4(h + 2a)}$ ($h^2 \gtrless 3a^2$ [illegible])

[illegible] $x_3 > 0$ [illegible] $h^2 > 3a^2 \Rightarrow h > \sqrt{3}a$
[illegible] $h = \sqrt{3}a$ [illegible]
[illegible]

08. (a) X [illegible] Y [illegible] $X \cup Y = S$
[illegible]
[illegible] $P(X \cup Y) = P(S) = 1$ [illegible]
X [illegible] Y [illegible]
[illegible]
$X \cap Y = \varnothing$ ([illegible]) [illegible]
[illegible] $P(X \cap Y) = p(\varnothing) = 0$ [illegible]
X [illegible] Y [illegible]
[illegible]
[illegible]

[illegible] $P(X \cap Y) = P(X) \times P(Y)$

A [illegible] B [illegible]
[illegible]
$P(A) + P(B) = 1$ [illegible]
$P(A \cap B) = 0$ [illegible]

$P(A) \frac{2}{5} =$ [illegible] $P(B) = \frac{3}{5}$ [illegible]

A [illegible] C [illegible] $P(C) = \frac{1}{2}$ [illegible]

(i) $P(A \cap C) = P(A) \cdot P(C)$

$= \frac{2}{5} \times \frac{1}{2}$

$= \frac{1}{5}$

(ii) $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$= \frac{2}{5} + \frac{1}{2} - \frac{1}{5}$

$= \frac{7}{10}$

$(A \cup C)$ [illegible] $(\overline{A \cup C})$ [illegible]
[illegible] $\overline{A \cup C} = \overline{A} \cap \overline{C}$ [illegible]

$P(\overline{A} \cap \overline{C}) = 1 - P(A \cup C)$

$= 1 - \frac{7}{10}$

$= \frac{3}{10}$

(iii) $P(A) = \frac{2}{5}$ [illegible] $P(\overline{A}) = 1 - \frac{2}{5} = \frac{3}{5}$ [illegible]

$P(C) = \frac{1}{2}$ [illegible] $P(\overline{C}) = 1 - \frac{1}{2} = \frac{1}{2}$ [illegible]

$P(\overline{A})\,P(\overline{C}) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10} = P(\overline{A} \cap \overline{C})$ [illegible] $\overline{A}$ [illegible] $\overline{C}$ [illegible]

$P(D) = \frac{1}{5}$ [illegible] $P(\overline{D}) = 1 - P(D) = 1 - \frac{1}{5} = \frac{4}{5}$

B [illegible] D [illegible]
$P(B \cup D) = P(B) + P(D)$ [illegible]

$P(B \cup D) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

$B \cup D$ [illegible] $\overline{B \cup D}$ ($\overline{B} \cap \overline{D}$) [illegible]
[illegible]

$P(\overline{B \cup D}) = P(\overline{B} \cap \overline{D}) = 1 - P(B \cup D) = 1 - \frac{4}{5} = \frac{1}{5} \neq 0$

[illegible] $\overline{B}$ [illegible] $\overline{D}$ [illegible]

(b) C : [illegible]
B : [illegible]
T : [illegible]
L : [illegible]
[illegible]

$P(C) = \frac{1}{10}$; $P(B) = \frac{2}{5}$; $P(T) = \frac{1}{2}$

$P(L/C) = \frac{1}{5}$, $P(L/B) = \frac{1}{2}$, $P(L/T) = \frac{3}{10}$ [illegible]

[illegible]

$$P(T/L) = \frac{P(L/T)\,P(T)}{P(L/C)\,P(C) + P(L/B)\,P(B) + P(L/T)\,P(T)}$$

$$= \frac{\frac{3}{10} \times \frac{1}{2}}{\left(\frac{1}{5} \times \frac{1}{10}\right) + \left(\frac{1}{2} \times \frac{2}{5}\right) + \left(\frac{3}{10} \times \frac{1}{2}\right)} = \frac{\frac{3}{20}}{\frac{1}{50} + \frac{2}{10} + \frac{3}{20}}$$

$= \frac{15}{37}$

09 (a) $\{x_1, x_2, \ldots, x_n\}$ [illegible] $\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$

[illegible] $\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}$

(i) $\{x_1 + \alpha, x_2 + \alpha, \ldots, x_n + \alpha\}$

$\therefore$ [illegible] $= \frac{1}{n}\sum_{i=1}^{n}(x_i + \alpha) = \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{1}{n}\sum_{i=1}^{n}\alpha$

$= \mu + \frac{1}{n}n\alpha = \mu + \alpha$ (α [illegible])

[illegible] $= \frac{1}{n}\sum_{i=1}^{n}[x_i + \alpha - (\mu + \alpha)]^2$

$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$

[illegible] $= \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2} = \sigma$

(ii) $\{\beta x_1, \beta x_2, \ldots, \beta x_n\}$

[illegible] $= \frac{1}{n}\sum_{i=1}^{n}\beta x_i$

$= \frac{\beta}{n}\sum_{i=1}^{n} x_i$ (β [illegible])

$= \beta\mu$

[illegible] $= \frac{1}{n}\sum_{i=1}^{n}(\beta x_i - \beta\mu)^2$

$= \frac{1}{n}\sum_{i=1}^{n}\beta^2(x_i - \mu)^2$ (β [illegible])

$= \beta^2\sigma^2$

$\therefore$ [illegible] $= \beta\sigma$

$\{2x_1+3,\ 2x_2+3, \dots 2x_n+3\}$ [illegible]

මධ්‍යන්‍යය $\mu' = \frac{1}{n}\sum_{i=1}^{n}(2x_i+3) = \frac{1}{n}\sum_{i=1}^{n} 2x_i + \frac{1}{n}\sum_{i=1}^{n} 3$

$$= 2\frac{1}{n}\sum_{i=1}^{n} x_i + \frac{1}{n}\, 3n$$

$$= 2\mu + 3$$

විචලතාව $\sigma'^2 = \frac{1}{n}\sum_{i=1}^{n} \{2x_i + 3 - (2\mu+3)\}^2$

$$= \frac{1}{n}\sum_{i=1}^{n} \{2x_i + \cancel{3} - 2\mu - \cancel{3}\}^2$$

$$= \frac{1}{n}4\sum_{i=1}^{n} (x_i - \mu)^2 = 4\sigma^2$$

$\therefore$ සම්මත අපගමනය $= 2\sigma$

(b) (i) 3, 6, 9, 12, 4, 6, 8, 10, 12, 14, x, y
3, 4, 6, 6, 8, 9, 10, 12, 12, 14, x, y

මධ්‍යන්‍යය $= \frac{3+4+6+6+8+9+10+12+12+14+x+y}{12}$

$= \frac{84+x+y}{12} = 8$ නිසා

$x + y = 12$

[illegible] 6 [illegible] x [illegible] y [illegible]
($\because$ 12 [illegible])
$\therefore$ $x = y = 6$

(ii) 3, 4, 6, 6, 6, 6, 8, 9, 10, 12, 12, 14

මධ්‍යස්ථය $= \frac{6+8}{2} = 7$

[illegible] $8-k,\ 8,\ 8+k$ [illegible]

[illegible] $\frac{8-k+8+8+k}{3} = 8$ වේ.

[illegible] 8 වේ.
[illegible] 12 නිසා

$$12 = \frac{1}{15}\sum_{i=1}^{n}(x_i - 8)^2$$

$$180 = (3-8)^2 + (4-8)^2 + 4(6-8)^2 + (8-8)^2 + (9-8)^2 + (10-8)^2 + 2(12-8)^2 + (14-8)^2 + (8-k-8)^2 + (8-8)^2 + (8+k-8)^2$$

$$180 = 25 + 16 + 16 + 0 + 1 + 4 + 32 + 36 + k^2 + 0 + k^2$$

$$180 = 2k^2 + 130$$

$$50 = 2k^2$$

$$k^2 = 25$$

$$k = \pm 5$$

*** ** ***