01. (a) α, β [illegible] $x^2 + bx + c = 0$ [illegible]

$\alpha + \beta = -b \qquad \alpha\beta = c$

[illegible] $\alpha^3\beta^2 + \alpha^2\beta^3 = \alpha^2\beta^2(\alpha + \beta) = -bc^2$

$\alpha^3\beta^2 \cdot \alpha^2\beta^3 = (\alpha\beta)^5 = c^5$

$\therefore \alpha^3\beta^2, \alpha^2\beta^3$ [illegible]

$x^2 - (\alpha^3\beta^2 + \alpha^2\beta^3)x + \alpha^3\beta^2 \cdot \alpha^2\beta^3 = 0$

[illegible] $x^2 + bc^2x + c^5 = 0$ //

$\alpha^3\beta^2 + \dfrac{1}{\alpha^2\beta^3} = \dfrac{\alpha^5\beta^5 + 1}{\alpha^2\beta^3} = \dfrac{c^5+1}{\alpha^2\beta^3}$

[illegible] $\alpha^2\beta^3 + \dfrac{1}{\alpha^3\beta^2} = \dfrac{c^5+1}{\alpha^3\beta^2}$

[illegible] $y = \dfrac{c^5+1}{x}$ [illegible]

[illegible]

$\left(\dfrac{c^5+1}{y}\right)^2 + bc^2\left(\dfrac{c^5+1}{y}\right) + c^5 = 0$

$(c^5+1)^2 + bc^2(c^5+1)y + c^5y^2 = 0$

$\therefore \alpha^3\beta^2 + \dfrac{1}{\alpha^2\beta^3}$ [illegible] $\alpha^2\beta^3 + \dfrac{1}{\alpha^3\beta^2}$

[illegible]

$c^5x^2 + bc^2(c^5+1)x + (c^5+1)^2 = 0$ [illegible]

(b) $f(x)$ [illegible] $(x - \alpha)$ [illegible] $\varnothing(x)$ [illegible] R [illegible]

$f(x) = \phi(x)(x-\alpha) + R$

$x = \alpha$ [illegible] $\underline{\underline{f(\alpha) = R}}$

α, β, γ [illegible]

$f(x)$ [illegible] $(x-\alpha)(x-\beta)(x-\gamma)$ [illegible] $A(x-\beta)(x-\gamma) + B(x-\alpha)(x-\gamma) + C(x-\alpha)(x-\beta)$ [illegible]

[illegible] $f(x) = \phi(x)(x-\alpha)(x-\beta)(x-\gamma)$
$+ A(x-\beta)(x-\gamma) + B(x-\alpha)(x-\gamma)$
$+ C(x-\alpha)(x-\beta)$
[illegible] $\phi(x)$ [illegible]

[illegible]

$f(\alpha) = A(\alpha-\beta)(\alpha-\gamma)$

$f(\beta) = B(\beta-\alpha)(\beta-\gamma)$ [illegible]

$f(\gamma) = C(\gamma-\alpha)(\gamma-\beta)$ [illegible]

[illegible] $A = \dfrac{f(\alpha)}{(\alpha-\beta)(\alpha-\gamma)}$, $B = \dfrac{f(\beta)}{(\beta-\alpha)(\beta-\gamma)}$ [illegible]

$C = \dfrac{f(\gamma)}{(\gamma-\alpha)(\gamma-\beta)}$ [illegible]

[illegible]

$\dfrac{f(\alpha)}{(\alpha-\beta)(\alpha-\gamma)}(x-\beta)(x-\gamma) + \dfrac{f(\beta)}{(\beta-\alpha)(\beta-\gamma)}(x-\alpha)(x-\gamma) + \dfrac{f(\gamma)}{(\gamma-\alpha)(\gamma-\beta)}(x-\alpha)(x-\beta)$ [illegible]

[illegible] x [illegible]

$-\left\{\dfrac{f(\alpha)}{(\alpha-\beta)(\alpha-\gamma)}(\beta+\gamma) + \dfrac{f(\beta)}{(\beta-\alpha)(\beta-\gamma)}(\alpha+\gamma) + \dfrac{f(\gamma)}{(\gamma-\alpha)(\gamma-\beta)}(\alpha+\beta)\right\}$

$f(x) = x^3 - kx$ [illegible] $\alpha = -1, \beta = 1$ [illegible] $\gamma = 2$ [illegible]

[illegible] x [illegible]

$-\left\{\dfrac{f(-1)}{(-1-1)(-1-2)}(1+2) + \dfrac{f(1)}{(1+1)(1-2)}(-1+2) + \dfrac{f(2)}{(2+1)(2-1)}(1+-1)\right\}$

$= -\left\{\dfrac{(-1)^3+k}{2} - \dfrac{(1)^3-k}{2}\right\}$

$= -k+1$

x [illegible]

$-k+1=0 \longrightarrow \underline{\underline{k=1}}$

02. (a)

P	H	O	I	L	S	Y
2	2	2	1	1	1	1

[illegible] 10 [illegible]

$= \dfrac{10!}{2!\,2!\,2!}$

$= 453600$

HISY [illegible]

[illegible] $= \dfrac{7!}{2!\,2!} = \dfrac{7 \times 6 \times 5 \times \cancel{4} \times 3 \times \cancel{2}}{\cancel{2} \times \cancel{2}}$

$= 1260$

$\therefore$ [illegible] H, I, S, Y [illegible]

[illegible] $= 4!$

$= 24$

$\therefore$ H [illegible] I, S, Y [illegible] H, I, S, Y [illegible]

[illegible] $= \dfrac{7!}{2!\,2!} \times 4!$

$= 1260 \times 24 = 30240$

H [illegible] I, S, Y [illegible]

[illegible] $= \dfrac{6!}{2!\,2!} \times 3!$

$= \dfrac{6 \times 5 \times 4 \times 3 \times \cancel{2} \times 3 \times \cancel{2}}{\cancel{2} \times \cancel{2}}$

$= 1080$

$\therefore$ [illegible] $= 30240 - 1080$

$= \underline{\underline{29160}}$

[illegible] 10 [illegible] 5 [illegible]

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(i) [illegible]

$= {}^3C_2 \times {}^5C_1 = 3 \times 5 = 15$

(ii) [illegible]

$= {}^3C_1 \times {}^5C_3 = 3 \times \dfrac{\cancel{5} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1}$

$= 60$

(iii) [illegible]

$= {}^7C_5 = \dfrac{7 \times 6}{2 \times 1}$

$= 21$

[illegible]

$= 15 + 60 + 21 = \underline{\underline{96}}$

(b) $P_n = n(n+1)(n+2) \ldots (n+r-1)$

[illegible] $n, r \in \mathbb{Z}^+$

$P_{n+1} = (n+1)(n+2)(n+3) \ldots (n+r)$

$nP_{n+1} = n(n+1)(n+2) \ldots (n+r)$

$= n(n+1)(n+2) \ldots (n+r-1)(n+r)$

$= n(n+1)(n+2) \ldots (n+r-1)n$
$+ n(n+1)(n+2) \ldots (n+r-1)r$

$= nP_n + rP_n$

[illegible] $P_{n+1} - P_n = r\left[\dfrac{P_n}{n}\right]$

$\dfrac{P_n}{n}$, $(r-1)!$ [illegible]

$r\left[\dfrac{P_n}{n}\right]$, $r!$ [illegible]

[illegible] $P_{n+1} - P_n$, $r!$ [illegible]

$P_{n+1} - P_n = K_n r!$ [illegible] $K_n \in \mathbb{Z}$

$P_1 = 1 \cdot 2 \cdot 3 \cdot 4 \ldots r = r!$

$P_2 - P_1 = K_1 r!$

$P_3 - P_2 = K_2 r!$

....

....

.... +

$P_{n-1} - P_{n-2} = K_{n-2} r!$

$P_n - P_{n-1} = K_{n-1} r!$

$P_n - P_1 = r! \sum_{r=1}^{n-1} K_r$

$P_n = r!\left(\sum_{r=1}^{n-1} K_r + 1\right)$

$\therefore P_n$, $r!$ [illegible]

$\therefore$ [illegible] r [illegible] $r!$ [illegible] //

03. (a) n [illegible]

$(x+y)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} y^r$.

${}^nC_r = \dfrac{n!}{(n-r)!\,r!}$ [illegible]

[illegible] $n = 1$ [illegible]

L.H.S $= x + y$

R.H.S $= {}^nC_0\, x + {}^nC_1\, y = x + y$

${}^nC_0 = {}^nC_1 = 1$ [illegible]

$\therefore n = 1$ [illegible]

$n = p$ [illegible]

[illegible] $p \in \mathbb{Z}^+$

[illegible] $(x+y)^p = \sum_{r=0}^{p} {}^pC_r\, x^{p-r} y^r$

[illegible] $(x+y)$ [illegible]

$(x+y)^{p+1} = \left(\sum_{r=0}^{p} {}^pC_r\, x^{p-r} y^r\right)(x+y)$

$= \sum_{r=0}^{p} {}^pC_r\, x^{p+1-r} y^r + \sum_{r=0}^{p} {}^pC_r\, x^{p-r} y^{r+1}$

[illegible] $r > 0$ සඳහා R.H.S. හි $x^{p+1-r}\, y^r$ හි සංගුණකය

$= {}^pC_r + {}^pC_{r-1}$ වේ.

$${}^pC_r + {}^pC_{r-1} = \frac{p!}{(p-r)!\,r!} + \frac{p!}{(p+1-r)!\,(r-1)!}$$

$$= \frac{p!}{(p-r)!\,(r-1)!}\left(\frac{1}{r} + \frac{1}{(p+1-r)}\right)$$

$$= \frac{p!}{(p-r)!\,(r-1)!}\left(\frac{p+1-\cancel{r}+\cancel{r}}{r\,(p+1-r)}\right)$$

$$= \frac{(p+1)!}{(p+1-r)!\,r!} = {}^{p+1}C_r \text{ සහ}$$

$${}^pC_0 = {}^pC_p = {}^{p+1}C_0 = {}^{p+1}C_{p+1} = 1 \text{ නිසා}$$

$$(x+y)^{p+1} = \sum_{r=0}^{p+1} {}^{p+1}C_r\, x^{p+1-r}\, y^r$$

$\therefore n = p$ විට සත්‍ය නම් $n = p+1$ විට ද ප්‍රතිඵලය සත්‍ය වේ.

[illegible] n සඳහා ප්‍රතිඵලය සත්‍ය වේ.

$$(x+y)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r}\, y^r \text{ හි } x = p,\ y = q$$

[illegible]

$$(p+q)^n = p^n + \sum_{r=1}^{n-1} {}^nC_r\, p^{n-r}\, q^r + q^n$$

$$(p+q)^n - p^n - q^n = \sum_{r=1}^{n-1} {}^nC_r\, p^{n-r}\, q^r$$

$$= pq \sum_{r=1}^{n-1} p^{n-r-1}\, q^{r-1}$$

n, p, q [illegible] Σ [illegible] $(p+q)^n - p^n - q^n$ [illegible] pq [illegible]

(b) $U_r = \dfrac{2r+1}{(3r-2)(3r+1)} \cdot \dfrac{1}{7^r}$

$$\frac{2r+1}{(3r-2)(3r+1)} = \frac{A}{3r-2} + \frac{B}{3r+1} \text{ ලෙස ගනිමු}$$

එවිට, $2r+1 = (3r+1)A + (3r-2)B$

$= 3(A+B)r + (A-2B)$

[illegible]

$A + B = \frac{2}{3}$ ——— ① සහ

$A - 2B = 1$ ——— ②

① - ② න් $3B = -\frac{1}{3} \Rightarrow B = -\frac{1}{9}$

$A = \frac{7}{9}$

එවිට $U_r = \dfrac{2r+1}{(3r-2)(3r+1)} \cdot \dfrac{1}{7^r}$

$$= \frac{1}{9}\left(\frac{7}{3r-2} - \frac{1}{3r+1}\right)\frac{1}{7^r}$$

$$= \frac{1}{9}\left(\frac{1}{3r-2}\cdot\frac{1}{7^{r-1}} - \frac{1}{3r+1}\cdot\frac{1}{7^r}\right)$$

දැන් $f(r) = \dfrac{1}{9}\cdot\dfrac{1}{(3r+1)}\cdot\dfrac{1}{7^r}$ ලෙස ගත් විට,

$U_r = f(r-1) - f(r)$ වේ.

$r = 1$ විට $U_1 = f(0) - \cancel{f(1)}$

$r = 2$ විට $U_2 = \cancel{f(1)} - \cancel{f(2)}$

$r = 3$ විට $U_3 = \cancel{f(2)} - \cancel{f(3)}$

$r = n-2$ විට $U_{n-2} = \cancel{f(n-3)} - \cancel{f(n-2)}$

$r = n-1$ විට $U_{n-1} = \cancel{f(n-2)} - \cancel{f(n-1)}$

$r = n$ විට $U_n = \cancel{f(n-1)} - f(n)$

$$S_n = \sum_{r=1}^{n} U_r = f(0) - f(n)$$

$$S_n = \frac{1}{9} - \frac{1}{9(3n+1)}\cdot\frac{1}{7^n}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{9} - \frac{1}{9(3n+1)}\cdot\frac{1}{7^n}\right)$$

$$= \frac{1}{9} \quad \because \lim_{n\to\infty} \frac{1}{9(3n+1)}\cdot\frac{1}{7^n} = 0$$

04. **(a)** $-80 - 18i$ හි වර්ගමූලයක් $a + ib$ යයි ගනිමු. මෙහි a හා b තාත්වික සංඛ්‍යා වේ.

එවිට,

$$(a+ib)^2 = -80 - 18i$$

$$\Rightarrow a^2 - b^2 + i\,2ab = -80 - 18i$$

තාත්වික හා අතාත්වික කොටස් [illegible]

$a^2 - b^2 = -80$ ——— ①

$2ab = -18$ ——— ②

② න් $b = -\frac{9}{a}$ එවිට ① න් $a^2 - \frac{81}{a^2} = -80$

$a^4 + 80a^2 - 81 = 0$

$(a^2 + 81)(a^2 - 1) = 0 \Rightarrow a^2 = -81$ හෝ $a^2 = 1$

a තාත්වික නිසා $a^2 \neq -81 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

එවිට $b = -9$ හෝ 9

$\therefore -80 - 18i$ හි වර්ගමූල $1 - 9i$ හා $-1 + 9i$ වේ.

$4z^2 + (16i - 4)z + (65 + 10i) = 0$ හි විවේචකය

$$(16i - 4)^2 - 16(65 + 10i) = (-256 - 128i + 16) - (1040 + 160i)$$

$$= -1280 - 288i$$

$$= 16(-80 - 18i)$$

$$= [4(1 - 9i)]^2$$

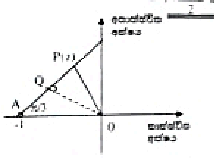
$(-80 - 18i = (1 - 9i)^2$ නිසා$)$

දී ඇති වර්ගජ සමීකරණයේ මූල $\dfrac{-(16i-4) \pm 4(1-9i)}{8}$

$$= \frac{8 - 52i}{8} \text{ සහ } \frac{20i}{8}$$

$$= 1 - \frac{13i}{2} \text{ සහ } \frac{5i}{2}$$

(b)



$\arg(z+1) = \frac{\pi}{3}$ [illegible] A [illegible] $\frac{\pi}{3}$ [illegible] P [illegible] OP [illegible]

[illegible] OP [illegible]

[illegible] $= OQ$

$= 1\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(c) $z^3 - 1 = (z - 1)(z^2 + z + 1)$

$z^3 - 1 = 0$ විට $(z - 1)(z^2 + z + 1) = 0$

[illegible] $z^2 + z + 1 = 0$ [illegible]

$\therefore z^3 - 1 = 0$ හි සංකීර්ණ මූල $\dfrac{-1 \pm \sqrt{3}i}{2}$ වේ.

දැන් $\omega = \dfrac{-1 + \sqrt{3}i}{2}$ යයි ගනිමු.

$$\text{එවිට, } \omega^2 = \left(\frac{-1 + \sqrt{3}i}{2}\right)^2$$

$$= \frac{1 - 3 - 2\sqrt{3}i}{4} = \frac{-1 - \sqrt{3}i}{2}$$

$$\text{එමෙන්ම } \left(\frac{-1 - \sqrt{3}i}{2}\right)^2 = \frac{1 - 3 + 2\sqrt{3}i}{2} = \frac{-1 + \sqrt{3}i}{2}$$

$\therefore$ [illegible] ω [illegible] ω^2 [illegible]

$z^3 - 1 = 0$ හි මූල $1, \omega, \omega^2$ නිසා

$\omega^2 + \omega + 1 = 0 \Rightarrow \omega + 1 = -\omega^2$

එවිට k [illegible]

$(\omega + 1)^k = (-\omega^2)^k$ [illegible]

k [illegible] $(-1)^k = -1$ නිසා

$(\omega + 1)^k = -\omega^{2k}$ වේ.

[illegible] $(\omega + 1)^k + \omega^{2k} = 0$

$f(x) = (x + 1)^k + x^{2k}$ යයි ගනිමු. මෙහි k [illegible]

එවිට, $f(\omega) = (\omega + 1)^k + \omega^{2k} = 0$

$f(\omega^2) = (\omega^2 + 1)^k + \omega^{4k}$

$= (-\omega)^k + \omega^{4k}$

$= (\omega^2)^k\,\omega^k - \omega^k$

$= \omega^k(\omega^{3k} - 1) \quad \omega^3 = 1$ නිසා

$= 0$

$\therefore (x - \omega)$ හා $(x - \omega^2)$, $f(x)$ හි සාධක වේ.

එනම්, $(x - \omega)(x - \omega^2) = x^2 - (\omega + \omega^2)x + \omega^3$

$= x^2 + x + 1$

$f(x)$ හි සාධකයක් වේ.

එනම් $x^2 + x + 1$, $(x + 1)^k + x^{2k}$ හි සාධකයකි. මෙහි k [illegible]

05. **(a)**

$$\lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x\to 0} \frac{\sin(x + \Delta x) - \sin x}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{2\cos\left(x + \frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\left(\frac{\Delta x}{2}\right)} \cdot \cos\left(x + \frac{\Delta x}{2}\right)$$

$$= \lim_{\Delta x\to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\left(\frac{\Delta x}{2}\right)} \cdot \lim_{\Delta x\to 0} \cos\left(x + \frac{\Delta x}{2}\right)$$

$$= 1 \cdot \cos x \quad \left(\lim_{\Delta x\to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\left(\frac{\Delta x}{2}\right)} = 1\right)$$

$$= \underline{\cos x}$$

$g(x) = \cos x$

$= \sin(\frac{\pi}{2} - x) \quad = f(\frac{\pi}{2} - x)$

$= f(y)$ මෙහි $y = \frac{\pi}{2} - x$

දැන් $\frac{d}{dx} g(x) = \frac{d}{dx} f(y) = \frac{d f(y)}{dy} \cdot \frac{dy}{dx}$

$= \cos y . (-1)$

$= \cos(\frac{\pi}{2} - x)$

$= \underline{\sin x}$

(i) $\frac{d}{dx} \sin(\ln(1 + x^2)) = \cos(\ln(1 + x^2)) . \frac{1}{1 + x^2} . 2x$

$= \frac{2x}{1 + x^2} \cos(\ln(1 + x^2))$

(ii) $\frac{d}{dx} \cos(\sin x) = -\sin(\sin x) . \cos x$

$= -\cos x \sin(\sin x)$

(b) (i) $y = \sin k\theta \operatorname{cosec}\theta$ සහ $x = \cos\theta$, k නියතයකි

$\frac{dy}{d\theta} = k\cos k\theta \operatorname{cosec}\theta - \sin k\theta . \operatorname{cosec}\theta \cot\theta$

සහ $\frac{dx}{d\theta} = -\sin\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$

$= -(k\cos k\theta \operatorname{cosec}\theta - \sin k\theta \operatorname{cosec}\theta \cot\theta) . \frac{1}{\sin\theta}$

$= -k\cos k\theta \operatorname{cosec}^2\theta + \sin k\theta \operatorname{cosec}^2\theta \cot\theta$

$\sin^2\theta \frac{dy}{dx} = -k\cos k\theta + \sin k\theta \cot\theta$

$(1 - x^2)\frac{dy}{dx} = -k\cos k\theta + xy$

$(1 - x^2)\frac{dy}{dx} - xy + k\cos k\theta = 0$

(ii) ඉහත සමීකරණය x විෂයෙන් අවකලනයෙන්,

$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - y - x\frac{dy}{dx} + k\frac{d}{d\theta}\cos k\theta \frac{d\theta}{dx} = 0$

$(1 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y - k^2 \sin k\theta \left(\frac{-1}{\sin\theta}\right) = 0$

$(1 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y + k^2 \sin k\theta \operatorname{cosec}\theta = 0$

$(1 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + (k^2 - 1)y = 0$

(c) $y(1 + x^2) = 2 \longrightarrow$ ①, x විෂයෙන් අවකලනයෙන්

$\frac{dy}{dx}(1 + x^2) + 2xy = 0$

$\frac{dy}{dx} = \frac{-2xy}{1 + x^2}$ = ඕනෑම ලක්ෂ්‍යයක දී වක්‍රයේ අනුක්‍රමණය

$P(3, \frac{1}{5})$ දී වක්‍රයට ඇඳි ස්පර්ශකයේ අනුක්‍රමණය

$= \frac{-2.3.\frac{1}{5}}{1 + 9}$

$= \frac{-3}{25}$

∴ P හි දී වක්‍රයට ඇඳි ස්පර්ශකයේ සමීකරණය

$y - \frac{1}{5} = \frac{-3}{25}(x - 3)$

$25y - 5 = -3x + 9$

$\Rightarrow 3x + 25y = 14 \longrightarrow$ ②

වක්‍රය ස්පර්ශකය හා නැවත හමුවේ නම්.

① හා ② න්

$\frac{2}{1 + x^2} = \frac{14 - 3x}{25}$

$50 = 14 + 14x^2 - 3x - 3x^3$

$\Rightarrow 3x^3 - 14x^2 + 3x + 36 = 0$

මෙහි මූල මගින් වක්‍රයේ P හි ස්පර්ශකය හමුවන ලක්ෂ්‍යවල x ඛණ්ඩාංක ලැබේ. x = 3 දී ස්පර්ශ වන බැවින් $(x - 3)^2$ සාධකයකි

$\therefore 3x^3 - 14x^2 + 3x + 36 = (x - 3)^2(3x + 4)$

∴ x = 3 (දෙවරක්) හා $x = -\frac{4}{3}$ ඉහත සමීකරණයේ මූල වේ

∴ P හි දී ඇඳි ස්පර්ශකය වක්‍රය නැවත හමුවන ලක්ෂ්‍යය Q බැවින්,

$Q = (\frac{-4}{3}, \frac{18}{25})$ වේ.

06. (a) $I_k = \int \frac{e^t}{t^k} dt = \int \frac{(-1)}{k-1} e^t \frac{d}{dt}\left(\frac{1}{t^{k-1}}\right) dt$

$= -\frac{1}{k-1}\frac{e^t}{t^{k-1}} + \frac{1}{k-1}\int \frac{e^t}{t^{k-1}} dt$

$(k-1)I_k - I_{k-1} + \frac{e^t}{t^{k-1}} = C \longrightarrow$ ①

C අභිමත නියතයකි

$I = \int e^x \left(\frac{1 - x}{1 + x}\right)^2 dx$

t = 1 + x ලෙස ගනිමු.

එවිට $I = \int \frac{e^{t-1}(2 - t)^2 dt}{t^2} = \frac{1}{e}\int e^t \left\{\frac{4}{t^2} - \frac{4}{t} + 1\right\} dt$

$I_k = \int \frac{e^t}{t^k} dt$ ලෙසින්

$I = \frac{1}{e}\{4I_2 - 4I_1 + \int e^t dt\}$

① න් $I_2 - I_1 = \frac{-e^t}{t} + C$

$\therefore I = \frac{1}{e}\left(\frac{-4e^t}{t} + 4C + e^t + D\right)$

මෙහි D අභිමත නියතයකි

$= \frac{e^{1+x}}{e} - \frac{4}{e}\frac{e^{1+x}}{(1+x)}$ + නියතයක්

(b) $J = \int_0^a f(x)\,dx$, a > 0 යැයි ගනිමු

$y = a - x$ නම් එවිට $dy = -dx$

එවිට $J = \int_a^0 f(a - y)(-dy)$

$= \int_0^a f(a - y)\,dy$

නිශ්චිත අනුකලය විචල්‍යයෙන් ස්වායත්ත බැවින්

$J = \int_0^a f(a - x)\,dx$

ඉහත ප්‍රතිඵලය අනුව,

$\int_0^{\pi/2} \sin^{2k} x\,dx = \int_0^{\pi/2} \sin^{2k}(\frac{\pi}{2} - x)\,dx$

$= \int_0^{\pi/2} \cos^{2k} x\,dx$

k ධන නිඛිලයකි

$I = \int_0^{\pi/2} \frac{\sin^{2k} x}{\sin^{2k} x + \cos^{2k} x}\,dx$

$= \int_0^{\pi/2} \frac{\cos^{2k} x}{\sin^{2k} x + \cos^{2k} x}\,dx$

$2I = \int_0^{\pi/2} \frac{\sin^{2k} x + \cos^{2k} x}{\sin^{2k} x + \cos^{2k} x}\,dx = \int_0^{\pi/2} dx$

$I = \frac{1}{2}[x]_0^{\pi/2} = \underline{\frac{\pi}{4}}$

07. ax + by + c = 0 රේඛාවට ලම්බ රේඛාවක සමීකරණය bx - ay + c' = 0 ආකාර වේ.

මෙම රේඛාව (x_0, y_0) හරහා යන විට,

$bx_0 - ay_0 + c' = 0$

∴ (x_0, y_0) හරහා යන ax + by + c = 0 ට ලම්බ රේඛාවේ සමීකරණය,

$bx - ay - bx_0 + ay_0 = 0$ වේ

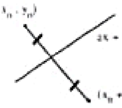
එනම්, $b(x - x_0) = a(y - y_0)$

$\Rightarrow \frac{x - x_0}{a} = \frac{y - y_0}{b} = t$ (යැයි ගනිමු)

එවිට, $x = x_0 + at \qquad y = y_0 + bt$

∴ (x_0, y_0) හරහා ax + by + c = 0 රේඛාවට ඇඳි ලම්බ රේඛාව මත පිහිටි ලක්ෂ්‍යයක් t පරාමිතියෙන් යුතු

$(x_0 + at, y_0 + bt)$ ආකාරයට ලිවිය හැක.



ax + by + c = 0 ඔස්සේ (x_0, y_0) හි දර්පණ ප්‍රතිබිම්බය $(x_0 + at, y_0 + bt)$ නම්.

එම ලක්ෂ්‍ය දෙකේ මධ්‍ය ලක්ෂ්‍යය වන $(x_0 + \frac{at}{2}, y_0 + \frac{bt}{2})$

ax + by + c = 0 මත වේ.

එවිට $a(x_0 + \frac{at}{2}) + b(y_0 + \frac{bt}{2}) + c = 0$

එමගින් $t = -2\frac{(ax_0 + by_0 + c)}{a^2 + b^2}$ බව ලැබේ.

∴ ax + by + c = 0 ඔස්සේ (x_0, y_0) හි දර්පණ ප්‍රතිබිම්බය

$(x_0 + at, y_0 + bt)$ වේ. මෙහි $t = \frac{-2(ax_0 + by_0 + c)}{a^2 + b^2}$ වේ.

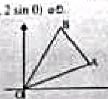
OA රේඛා ඛණ්ඩයේ ලම්බ සමච්ඡේදකය

$x\cos\theta + y \sin\theta - 1 = 0$ බැවින්, A යනු

$x\cos\theta + y\sin\theta - 1 = 0$ ඔස්සේ O හි දර්පණ ප්‍රතිබිම්බය යි.

∴ A හි ඛණ්ඩාංක $(x_0 + at, y_0 + bt)$ මගින්

$(2\cos\theta, 2\sin\theta)$ වේ.



$\therefore$ OA රේඛාවේ සමීකරණය $\frac{y}{x} = m$ මගින්

$$\frac{y}{x} = \frac{\cancel{2}\sin\theta}{\cancel{2}\cos\theta}$$

$y = x\tan\theta$ වේ.//

AB හි ලම්බ සමච්ඡේදකය

$x - y - 1 = 0$ සැලකීමෙන් B යනු $x - y - 1 = 0$ තුළින් A හි ප්‍රතිබිම්බය වේ.

$\therefore$ B හි ඛණ්ඩාංක $\left\{2\cos\theta - \cancel{2}\frac{(2\cos\theta - 2\sin\theta - 1)}{\cancel{2}},\ 2\sin\theta + \cancel{2}\frac{(2\cos\theta - 2\sin\theta - 1)}{\cancel{2}}\right\}$

$= (1 + 2\sin\theta,\ -1 + 2\cos\theta)$ වේ.

$\therefore$ OB රේඛාවේ සමීකරණය $\frac{y}{x} = m$ මගින්

$$\frac{y}{x} = \frac{-1 + 2\cos\theta}{1 + 2\sin\theta}$$

$$y(1 + 2\sin\theta) = x(-1 + 2\cos\theta)$$

එනම්, $x(1 - 2\cos\theta) + y(1 + 2\sin\theta) = 0$

AB හි සමීකරණය

$$\frac{y - 2\sin\theta}{x - 2\cos\theta} = \frac{-1 + 2\cos\theta - 2\sin\theta}{1 + 2\sin\theta - 2\cos\theta}$$

OB හි ලම්බ සමච්ඡේදකයේ සමීකරණය

$$y - \left(\cos\theta - \frac{1}{2}\right) = \frac{-(1 + 2\sin\theta)}{(-1 + 2\cos\theta)}\left\{x - \left(\sin\theta + \frac{1}{2}\right)\right\}$$

$$y(2\cos\theta - 1) + x(1 + 2\sin\theta) - \frac{1}{2}(2\cos\theta - 1)^2 - \frac{1}{2}(2\sin\theta + 1)^2 = 0$$

$$y(2\cos\theta - 1) + x(1 + 2\sin\theta) - \frac{1}{2}\{4\cos^2\theta - 4\cos\theta + 1 + 4\sin^2\theta + 4\sin\theta + 1\} = 0$$

$$y(2\cos\theta - 1) + x(1 + 2\sin\theta) - \frac{1}{2}(6 - 4\cos\theta + 4\sin\theta) = 0$$

$$y(2\cos\theta - 1) + x(2\sin\theta + 1) - (3 - 2\cos\theta + 2\sin\theta) = 0 \quad \text{——— ①}$$

OA හා AB රේඛාවල ලම්බ සමච්ඡේදක වල ඡේදන ලක්ෂ්‍ය වල ඛණ්ඩාංක $x\cos\theta + y\sin\theta - 1 = 0$ හා $x - y - 1 = 0$ සමීකරණ දෙක මගින් ලැබේ.

එය විසඳීමෙන්,

$$x = \frac{1 + \sin\theta}{\cos\theta + \sin\theta} \text{ හා } y = \frac{1 - \cos\theta}{\cos\theta + \sin\theta}$$

$\therefore$ OA හා AB හි ලම්බ සමච්ඡේදක වල ඡේදන ලක්ෂ්‍යය

$$\left(\frac{1 + \sin\theta}{\cos\theta + \sin\theta},\ \frac{1 - \cos\theta}{\cos\theta + \sin\theta}\right)$$

① න් $\frac{(1 - \cos\theta)}{\cos\theta + \sin\theta}(2\cos\theta - 1) + \frac{(1 + \sin\theta)}{(\cos\theta + \sin\theta)}(2\sin\theta + 1)$

$$= \frac{3\cos\theta - \cancel{1} - 2\cos^2\theta + 3\sin\theta + 2\sin^2\theta + \cancel{1}}{\cos\theta + \sin\theta}$$

$$= \frac{3(\cos\theta + \sin\theta) + 2(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\cos\theta + \sin\theta}$$

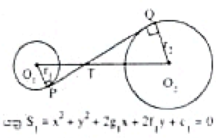
$$= \frac{\cancel{(\cos\theta + \sin\theta)}(3 + 2\sin\theta - 2\cos\theta)}{\cancel{\cos\theta + \sin\theta}}$$

$$= (3 + 2\sin\theta - 2\cos\theta)$$

$\therefore \left(\frac{1 + \sin\theta}{\cos\theta + \sin\theta},\ \frac{1 - \cos\theta}{\cos\theta + \sin\theta}\right)$ ලක්ෂ්‍යය OB හි ලම්බ සමච්ඡේදකය මත ද පිහිටයි.

එනම් OA, AB හා OB හි ලම්බ සමච්ඡේදක එක ලක්ෂ්‍යයක වේ.

08.



PQ යනු $S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

හා $S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

වෘත්ත දෙකට ඇඳි පොදු ස්පර්ශකය යයි ද එය O_1O_2 රේඛා ඛණ්ඩය O_1 හා O_2 අතර T ලක්ෂ්‍යයේ දී ඡේදනය කරන්නේ යයි සිතමු.

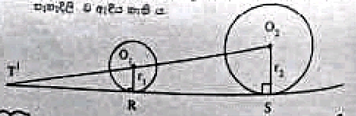
එවිට, $O_1PT\,\Delta$ හා $O_2QT\,\Delta$ සමරූපී වේ.

$\therefore \frac{O_1T}{TO_2} = \frac{r_1}{r_2}$ මෙහි r_1 හා r_2 පිළිවෙළින් $S_1 = 0$ හා $S_2 = 0$ හි අරයන් වේ.

T යනු O_1O_2 රේඛා ඛණ්ඩය $r_1 : r_2$ අනුපාතයට අභ්‍යන්තරව බෙදන ලක්ෂ්‍යය බැවින්,

$$T = \left(\frac{-g_1r_2 - g_2r_1}{r_1 + r_2},\ \frac{-f_1r_2 - f_2r_1}{r_1 + r_2}\right)$$

මෙලෙස T හරහා තවත් පොදු ස්පර්ශකයක් වෘත්ත දෙකට ඇඳිය හැකි ය.



$S_1 = 0$ හා $S_2 = 0$ ට ඇඳි පොදු ස්පර්ශකයක් RS යයි ද එය O_1O_2 රේඛාව දික් කළ විට T^1 හි දී හමුවන්නේ යයි ගනිමු.

එවිට T^1RO_1 හා T^1SO_2 සමරූපී බැවින්,

$$\frac{O_1T^1}{O_2T^1} = \frac{r_1}{r_2}$$

එනම්, T^1 යනු O_1O_2 රේඛාව බාහිරව $r_1 : r_2$ අනුපාතයට බෙදන ලක්ෂ්‍යය යි.

$$T^1 = \left(\frac{g_1r_2 - g_2r_1}{r_1 - r_2},\ \frac{f_1r_2 - f_2r_1}{r_1 - r_2}\right)$$

පැහැදිලිව ම T^1 හරහා තවත් පොදු ස්පර්ශකයක් වෘත්ත දෙකට ඇඳිය හැකි ය.

$x^2 + y^2 - 18x + 6y + 86 = 0$ හා $x^2 + y^2 + 18x - 6y + 74 = 0$ වෘත්ත දෙකේ සමීකරණ

$(x - 9)^2 + (y + 3)^2 = 2^2$ හා $(x + 9)^2 + (y - 3)^2 = 4^2$

ආකාරයට ලිවිය හැක.

එනම් T හා T^1 හි ඛණ්ඩාංක පිළිවෙළින්,

$$\left(\frac{9 \times 4 - 9 \times 2}{2 + 4},\ \frac{-3 \times 4 + 3 \times 2}{2 + 4}\right) = (3, -1) \text{ හා}$$

$$\left(\frac{-2 \times 9 + 4 \times (-9)}{2 - 4},\ \frac{-2 \times (-3) + 4 \times 3}{2 - 4}\right) = (27, -9)$$

වේ.

T හරහා ඇඳි රේඛාවක සමීකරණය $y + 1 = m(x - 3)$ ආකාර වේ.

එනම්, $y - mx + 3m + 1 = 0$ වේ.

මෙම රේඛාව $(x - 9)^2 + (y + 3)^2 = 2^2$ ස්පර්ශ කරයි නම්,

$$\frac{|-3 - 9m + 3m + 1|}{\sqrt{1 + m^2}} = 2$$

$$\begin{aligned}
\text{එනම්, } (-6m - 2)^2 &= 4(1 + m^2)\\
\cancel{4}(3m + 1)^2 &= \cancel{4}(1 + m^2)\\
8m^2 + 6m &= 0\\
4m^2 + 3m &= 0\\
m = 0 \text{ හෝ } m &= \frac{-3}{4}
\end{aligned}$$

මෙලෙස T හරහා යන පොදු ස්පර්ශක වල සමීකරණ

$y + 1 = 0$ හා $y + 1 = -\frac{3}{4}(x - 3)$

එනම් $y + 1 = 0$ හා $4y + 3x - 5 = 0$

T^1 හරහා යන රේඛාවක සමීකරණය

$y + 9 = m'(x - 27)$ ආකාර වේ.

එනම්, $y - m'x + 27m' + 9 = 0$

මෙම රේඛාව $(x - 9)^2 + (y + 3)^2 = 2^2$ වෘත්තය ස්පර්ශ කරන විට,

$$\frac{|-3 - 9m' + 27m' + 9|}{\sqrt{1 + m'^2}} = 2$$

$$\begin{aligned}
(6 + 18m')^2 &= 4(1 + m'^2)\\
36(1 + 3m')^2 &= 4(1 + m'^2)\\
9(1 + 6m' + 9m'^2) &= 1 + m'^2\\
9 + 54m' + 81m'^2 &= 1 + m'^2\\
80m'^2 + 54m' + 8 &= 0\\
40m'^2 + 27m' + 4 &= 0\\
m' &= \frac{-27 \pm \sqrt{27^2 - 640}}{80}\\
&= \frac{-27 \pm \sqrt{729 - 640}}{80}\\
&= \frac{-27 \pm \sqrt{89}}{80}
\end{aligned}$$

මෙලෙස T^1 හරහා වෘත්ත දෙකට ඇඳි පොදු ස්පර්ශක වල සමීකරණ

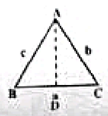
$$y + 9 = \frac{-27 + \sqrt{89}}{80}(x - 27) \text{ හා}$$

$$y + 9 = \frac{-27 - \sqrt{89}}{80}(x - 27) \text{ වේ.}$$

09. ABC ත්‍රිකෝණයක සයින් නීතිය අනුව

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \text{ වේ.}$$

සාධනය :-

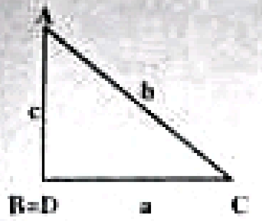


(i) ABC සුළුකෝණී ත්‍රිකෝණයක් විට

$$\begin{aligned}
AD &= AB\sin B\\
&= AC\sin C
\end{aligned}$$

$$\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$$

(ii)



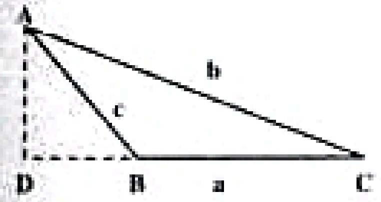
ABC සෘජුකෝණී ත්‍රිකෝණයෙන් විට

$AD = AB = AC \sin C$

$AB \sin B = AC \sin C \ (\because B = 90^0)$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

(iii)



ABC මහාකෝණී ත්‍රිකෝණයෙන් විට

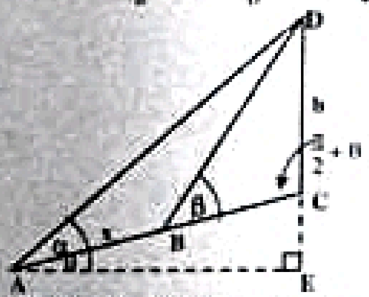
$AD = AB \sin(\pi - B)$

$= AC \sin C$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

එමෙන්ම, $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \ //$$



ACD ත්‍රිකෝණයට සයින් සූත්‍රය යෙදීමෙන්

$$\frac{AD}{\sin(\frac{\pi}{2} + \theta)} = \frac{h}{\sin\alpha}$$

$$\Rightarrow h = \frac{AD \sin\alpha}{\cos\theta}$$

ABD ත්‍රිකෝණයට සයින් සූත්‍රය යෙදීමෙන්

$$\frac{AD}{\sin(\pi - \beta)} = \frac{x}{\sin(\beta - \alpha)} \Rightarrow AD = \frac{x \sin\beta}{\sin(\beta - \alpha)}$$

$$\therefore h = \frac{x \sin\beta \sin\alpha}{\sin(\beta - \alpha)\cos\theta}$$

(iii) $d = DE = AD \sin(\alpha + \theta)$ හා

$AD = \dfrac{x \sin\beta}{\sin(\beta - \alpha)}$ බැවින්

$$d = \frac{x \sin\beta \sin(\alpha + \theta)}{\sin(\beta - \alpha)}$$

(b) (i) $\sin\theta - \cos\theta = 1$

$$\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\sin\theta\cos\frac{\pi}{4} - \cos\theta\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\sin(\theta - \frac{\pi}{4}) = \sin\frac{\pi}{4}$$

මෙහි සාධාරණ විසඳුම,

$$\theta - \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}, \ n \in \mathbb{Z}$$

$$\theta = n\pi + (1 + (-1)^n)\frac{\pi}{4}$$

$n = 0$ විට $\theta = \dfrac{\pi}{2}$ හා $n = 1$ විට $\theta = \pi$

(ii) $\tan^{-1}\dfrac{1}{2} - \tan^{-1}\dfrac{1}{3} = \sin^{-1} x$

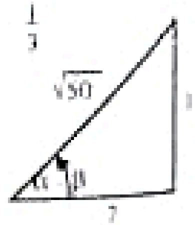
$\alpha = \tan^{-1}\dfrac{1}{2}$, $\beta = \tan^{-1}\dfrac{1}{3}$ නම්

$\tan\alpha = \dfrac{1}{2}$ හා $\tan\beta = \dfrac{1}{3}$ වේ

$$\text{එවිට, } \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$= \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2}\cdot\frac{1}{3}}$$

$$= \frac{1}{7}$$



$$\sin(\alpha - \beta) = \frac{1}{\sqrt{50}}$$

ඒනිසා $\alpha - \beta = \sin^{-1}\dfrac{1}{\sqrt{50}}$

නමුත් $\alpha - \beta = \sin^{-1} x$

$$\sin^{-1} x = \sin^{-1}\frac{1}{\sqrt{50}}$$

$$\therefore x = \frac{1}{\sqrt{50}} = \frac{5\sqrt{2}}{50} = \frac{\sqrt{2}}{10}$$

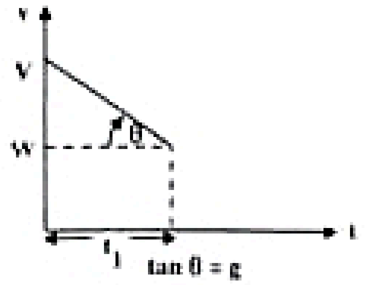
*****

8. (a) (i) $0 \le t \le t_1$ [illegible]

$(ඩ \,{}^{ප්‍ර}\, E) = \uparrow U \;;\; (ඩ \,{}^{ත්‍ව}\, E) = 0$

$(P \,{}^{ප්‍ර}\, ඩ) = \uparrow V \;;\; (P \,{}^{ත්‍ව}\, E) = \downarrow g$

$\tan\theta = \dfrac{V - W}{t_1} \rightarrow t_1 = \dfrac{V - W}{g} \quad \therefore W = V - gt_1$



$\tan\theta = g$

(ii) $t_1 \le t \le t_2$ [illegible]

$(P \,{}^{ත්‍ව}\, E) = \downarrow g \quad (Q \,{}^{ත්‍ව}\, E) = \downarrow g$

$(Q \,{}^{ත්‍ව}\, P) = (Q \,{}^{ත්‍ව}\, E) + (E \,{}^{ත්‍ව}\, P)$

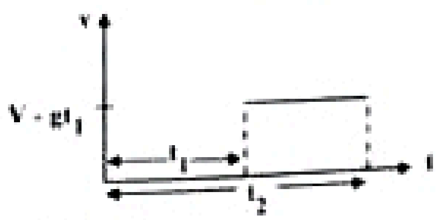
$= \downarrow g + \uparrow g = 0$

$\therefore (Q \,{}^{ප්‍ර}\, P)$ [illegible] $(Q \,{}^{ප්‍ර}\, P) = \uparrow (V - gt_1)$

$t = t_1$ [illegible]

$(P \,{}^{D}\, Q) = (P \,{}^{D}\, E) = \tfrac{1}{2}(V + W)t_1$

$= \tfrac{1}{2}(V + V - gt_1)t_1$



$t = t_1$ [illegible]

$(Q \,{}^{ප්‍ර}\, P) = (Q \,{}^{ප්‍ර}\, ඩ) + (ඩ \,{}^{ප්‍ර}\, P)$

$= \uparrow V + \downarrow W$

$= \uparrow V - W + \uparrow V - (V - gt_1)$

$= \uparrow gt_1$

$t_2 - t_1$ [illegible] $= gt_1(t_2 - t_1)$

$t = t_2$ [illegible] P [illegible] Q [illegible]

[illegible] $(Q \,{}^{D}\, P) = 0$ [illegible]

$gt_1(t_2 - t_1) - \dfrac{1}{2}(2V - 2gt_1)t_1 = 0$

$gt_1(t_2 - t_1) = \dfrac{t_1}{2}(2V - gt_1)$

$t_2 - t_1 = \dfrac{1}{2g}(2V - gt_1) \; (t_2 > t_1$ [illegible]$)$

$t_2 = \dfrac{V}{g} - \dfrac{t_1}{2} + t_1$

$= \dfrac{V}{g} + \dfrac{t_1}{2}$

$t = t_2$ [illegible]

$(Q \,{}^{ප්‍ර}\, E) = U + V - g(t_2 - t_1)$

$= U + V - g\left(\dfrac{V}{g} + \dfrac{t_1}{2} - t_1\right)$

$= U + \dfrac{gt_1}{2}$

$(P \,{}^{ප්‍ර}\, E) = (P \,{}^{ප්‍ර}\, Q) + (Q \,{}^{ප්‍ර}\, E)$

$= -gt_1 + U + \dfrac{gt_1}{2}$

$= U - \dfrac{gt_1}{2}$

(b) $(ස, E) = U \quad (ක, E) = \uparrow V$

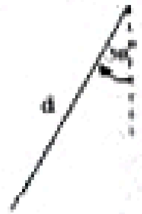
$(ස, ක) = (ස, E) + (E, ක)$

$= u + \downarrow v$

$= \downarrow v + u$

$= \overrightarrow{PQ} + \overrightarrow{QR}$

$= \overrightarrow{PR}$



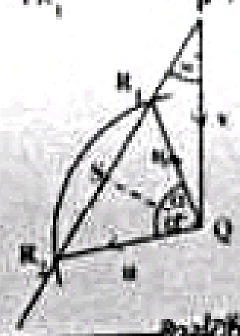
Q [illegible] $(u < v < 2u)$ [illegible] P [illegible] R_1 [illegible] R_2 [illegible] $\overrightarrow{QR_1}$ [illegible] $\overrightarrow{QR_2}$ [illegible]

[illegible] $R_1\hat{Q}R_2 = 2\alpha$ [illegible]

[illegible] $\cos\alpha = \dfrac{V\sin 30}{u} = \dfrac{V}{2u}$ [illegible]

[illegible] t_1 [illegible] t_2 [illegible]

$t_1 = \dfrac{d}{PR_1}$ [illegible] $t_2 = \dfrac{d}{PR_2}$ [illegible]



කාල වෙනස $t_1 - t_2 = \frac{d}{PR_1} - \frac{d}{PR_2}$

$$= d\left[\frac{PR_2 - PR_1}{PR_1 \cdot PR_2}\right]$$

$$= d\frac{[PS + SR_2 - (PS - SR_1)]}{(PS - SR_1)(PS + SR_2)}$$

$$= \frac{d \cdot 2SR_1}{PS^2 - SR_1^2}$$

($\because SR_1 = SR_2$ බැවින්)

$$t_1 - t_2 = \frac{2d\sqrt{u^2 - (v/2)^2}}{(v\sqrt{3}/2)^2 - [u^2 - (v/2)^2]}$$

$$= \frac{2d\,½\sqrt{4u^2 - v^2}}{¼(3v^2 - 4u^2 + v^2)}$$

$$= \frac{d\sqrt{4u^2 - v^2}}{v^2 - u^2}$$

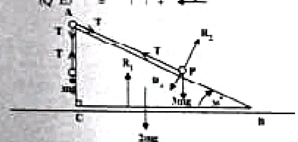
02. (a) (P, ෆ) = a_1 30°

(Q, ෆ) = $\uparrow a_1$

(ෆ, E) = $\leftarrow a_2$

(P, E) = a_1 30° + $\leftarrow a_2$

(Q, E) = $\uparrow a_1 + \leftarrow a_2$



$E = mg$ යොදයි.

$\uparrow$ Q : $T - mg = ma_1$ ——— ①

P ; $3mg\sin 30^\circ - T = 3m(a_1 - a_2\cos 30^\circ)$ ——— ②

$\leftarrow$ පද්ධතිය ; $0 = 2ma_2 + m.a_2 + 3m(a_2 - a_1\cos 30)$ ——— ③

① + ② $\frac{\cancel{m}g}{2} = 4\cancel{m}a_1 - 3\cancel{m}a_2\frac{\sqrt{3}}{2}$ ——— ④

③ න්, $0 = 6\cancel{m}a_2 - 3\cancel{m}a_1\frac{\sqrt{3}}{2}$ ——— ⑤

⑤ න් $a_1 = \frac{4a_2}{\sqrt{3}}$

④ න් $\frac{g}{2} = 4\times\frac{4a_2}{\sqrt{3}} - \frac{3\sqrt{3}a_2}{2}$

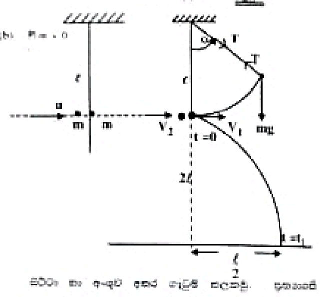
$\sqrt{3}g = 32a_2 - 9a_2$

$a_2 = \frac{\sqrt{3}g}{23}$

$\therefore a_1 = \frac{4}{\sqrt{3}}\times\frac{\sqrt{3}g}{23} = \frac{4g}{23}$

$T = mg + \frac{4mg}{23} = \frac{27mg}{23}$

(b) $t = 0$



ඝට්ටනය හා අංශුව අතර ගැටුම සලකමු. ගම්‍යතා සංස්ථිති නියමයෙන්,

$\cancel{m}u = \cancel{m}v_1 + \cancel{m}v_2$ ——— ①

නිව්ටන්ගේ පරීක්ෂණාත්මක නියමයෙන්,

$eu = v_1 - v_2$ ——— ②

① හා ② විසඳීමෙන්

$v_1 = \frac{u}{2}(1 + e)$ හා $v_2 = \frac{u}{2}(1 - e)$

අංශුවේ චලිතය සලකා $S = ut + \frac{1}{2}at^2$ යෙදීමෙන්

$\rightarrow \frac{\ell}{2} = v_2 t_1$ ——— ③

$\downarrow 2\ell = \frac{1}{2}g t_1^2$ ——— ④

t_1 ඉවත් කිරීමෙන් $16v_2^2 = g\ell$ ——— ⑤

චලිතය සලකා ශක්ති සංස්ථිති නියමය යෙදීමෙන්

$$\frac{1}{2}mv_1^2 - mg\ell = 0 - mg\ell\cos\alpha$$

$v_1^2 = 2g\ell(1 - \cos\alpha)$ ——— ⑥

$\frac{⑥}{⑤}$ න් $\frac{v_1^2}{16v_2^2} = \frac{2g\ell(1 - \cos\alpha)}{g\ell}$

$$\Rightarrow \frac{v_1^2}{v_2^2} = 32\times 2\sin^2\frac{\alpha}{2}$$

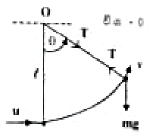
$\frac{v_1}{v_2} = \frac{1 + e}{1 - e}$ බැවින් $\frac{(1 + e)^2}{(1 - e)^2}$

$= 64\sin^2\frac{\alpha}{2}$

$\Rightarrow \frac{1 + e}{1 - e} = 8\sin\frac{\alpha}{2}$ ($\because 0 < e < 1$ නිසා)

$$\therefore e = \frac{8\sin\frac{\alpha}{2} - 1}{8\sin\frac{\alpha}{2} + 1}$$

03. අංශුවේ චලිතය සලකා ශක්ති සංස්ථිති නියමය යෙදීමෙන්.



$$\frac{1}{2}mu^2 - mg\ell = \frac{1}{2}mv^2 - mg\ell\cos\theta$$

$v^2 = u^2 - 2g\ell(1 - \cos\theta)$ ——— ①

$E = mg$ යෙදීමෙන්,

$T - mg\cos\theta = m\frac{v^2}{\ell}$ ——— ②

ආදේශයෙන්,

(i) $T = mg\cos\theta + \frac{m}{\ell}[u^2 - 2g\ell + 2g\ell\cos\theta]$

$= \frac{m}{\ell}[u^2 - 2g\ell + 3g\ell\cos\theta]$

$T = m[3g\cos\theta - 2g + \frac{u^2}{\ell}]$

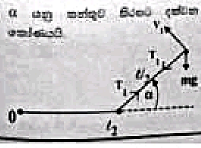
(ii) අංශුව යන්තමින් θ ... ළඟාවීමට නම්,

$\theta = \frac{\pi}{2}$ විට $v \geq 0$ විය යුතු ය

① න් $u^2 - 2g\ell \geq 0 \Rightarrow u^2 \geq 2g\ell$

$\therefore$ අවම $u = \sqrt{2g\ell}$

(iii) α යනු තන්තුව තිරස සමග සාදන ආනතිය කෝණයයි.



අංශුවේ චලිතය සලකා, ශ. සං. නි. යෙදීමෙන්,

$$\frac{1}{2}m(u^2 - 2g\ell) + 0 = \frac{1}{2}mv_1^2 + mg\frac{\ell}{2}\sin\alpha$$

$u^2 - 2g\ell = v_1^2 + g\ell\sin\alpha$

$v_1^2 = u^2 - 2g\ell - g\ell\sin\alpha$

$E = mg$ යෙදීමෙන්,

$T_1 + mg\sin\alpha = \frac{mv_1^2}{\ell/2}$

$T_1 = \frac{2m}{\ell}(u^2 - 2g\ell - g\ell\sin\alpha) - mg\sin\alpha$

තන්තුව බුරුල් වීමට නම් $T_1 = 0$

$\Rightarrow \frac{2m}{\ell}(u^2 - 2g\ell - g\ell\sin\alpha) - mg\ell\sin\alpha = 0$

$2u^2 - 4g\ell - 2g\ell\sin\alpha - g\ell\sin\alpha = 0$

$\sin\alpha = \frac{2u^2 - 4g\ell}{3g\ell}$ වේ.

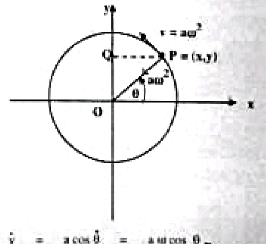
$0 < \sin\alpha < 1$ නිසා

$0 < \frac{2u^2 - 4g\ell}{3g\ell} < 1 \Rightarrow 2g\ell < u^2 < \frac{7g\ell}{2}$

04. $OQ = a\sin\theta$

$\therefore y = a\sin\theta$ ——— ①

t විෂයෙන් අවකලනයෙන්,



$\dot{y} = a\cos\theta\,\dot{\theta} = a\omega\cos\theta$ ($\because \dot{\theta} = \frac{d\theta}{dt} = \omega$)

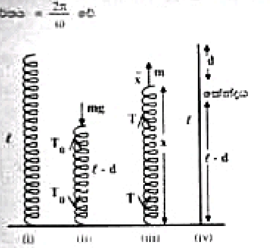
$\therefore \ddot{y} = -a\omega^2\sin\theta$

$\ddot{y} = -\omega^2 y$ (① න්)

එවිට $-a \leq y \leq a$ වේ.

∴ P හි y [illegible] Q [illegible]

∴ [illegible] $= \frac{2\pi}{\omega}$ වේ.



(ii) වන අවස්ථාව අනුව සමතුලිත පිහිටීමේ දී.

$$T_0 = mg \text{ ——— ①}$$

[illegible] $T_0 = \frac{\lambda d}{\ell}$ ——— ②

(ප්‍රත්‍යාස්ථතා මාපාංකය λ නම්)

$$\therefore mg = \frac{\lambda d}{\ell} \Rightarrow \frac{\lambda}{\ell} = \frac{mg}{d}$$

(iii) වන අවස්ථාව අනුව.

$$T = \frac{\lambda(\ell - x)}{\ell} = \frac{mg}{d}(\ell - x)$$

$F = ma$ යෙදීමෙන්. $\uparrow T - mg = m\ddot{x}$

$$\frac{\cancel{m}g}{d}(\ell - x) - \cancel{m}g = \cancel{m}\ddot{x}$$

$$-\frac{g}{d}(x - \ell + d) = \ddot{x}$$

[illegible] $x = 0$ [illegible] $x = \ell - d$ [illegible] m [illegible]

[illegible]

[illegible] a [illegible] t [illegible] y [illegible]

$$v^2 = \omega^2 (a^2 - y^2)$$

[illegible] h [illegible]

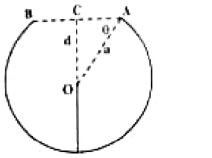
[illegible] v_1 [illegible] $v_1^2 = 2gh$ [illegible]

$y = d$ [illegible] $v = v_1$ [illegible]

$$2gh = \frac{g}{d}(a^2 - d^2) \Rightarrow a^2 = 2dh + d^2$$

∴ [illegible] $a = \sqrt{2dh + d^2}$ [illegible]

[illegible] $d \le a$ [illegible]
(iv [illegible])
$\ell \ge a + d$ [illegible]



[illegible]

$O\hat{A}C = \theta$ නම්

$$\tan\theta = \frac{d}{\sqrt{a^2 - d^2}}$$

[illegible] T නම්

$$T = 2\left(\frac{\pi/2 + \theta}{\omega}\right)$$

$$= 2\sqrt{\frac{d}{g}}\left(\frac{\pi}{2} + \tan^{-1}\frac{d}{\sqrt{a^2 - d^2}}\right)$$

$$= \sqrt{\frac{d}{g}}\left(\pi + 2\tan^{-1}\frac{d}{\sqrt{2dh + d^2 - d^2}}\right)$$

$$= \sqrt{\frac{d}{g}}\left(\pi + 2\tan^{-1}\sqrt{\frac{d}{2h}}\right)$$

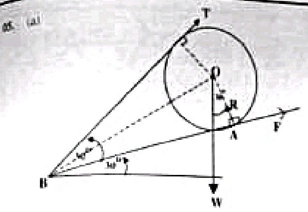
[illegible] $\frac{3\pi}{2}\sqrt{\frac{d}{g}}$ [illegible]

$$\sqrt{\frac{d}{g}}\left(\pi + 2\tan^{-1}\sqrt{\frac{d}{2h}}\right) \ge \frac{3\pi}{2}\sqrt{\frac{d}{g}} \Rightarrow \tan^{-1}\sqrt{\frac{d}{2h}} \ge \frac{\pi}{4}$$

$$\Rightarrow \sqrt{\frac{d}{2h}} \ge 1 \Rightarrow \frac{h}{d} \le \frac{1}{2}$$

∴ [illegible] $\frac{h}{d} = \frac{1}{2}$

05. (a)



[illegible] a [illegible] O [illegible]

$$F \times a = T \times a \text{ ——— ①}$$

A [illegible]

$$W \times a\,\cancel{\sin}\,30 = T \times \cancel{a}\tan 75 \times \cancel{\sin}\,30$$

$$W = T\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$$

$$W = T\frac{(4 + 2\sqrt{3})}{2}$$

$$T = \frac{W}{2 + \sqrt{3}}$$

$$T = W(2 - \sqrt{3}) = F \text{ (① [illegible])}$$

B $R \times \cancel{a}\tan 75 = W \times \cancel{a}\sec 75^\circ \times \cos 45^\circ$

$$R\frac{\sin 75}{\cancel{\cos 75}} = W\frac{1}{\cancel{\cos 75}} \times \frac{1}{\sqrt{2}}$$

$$R = \frac{W}{\sqrt{2}\sin 75} = \frac{W}{\sqrt{2}\cos 15}$$

[illegible] $\frac{F}{R} \le \mu$ [illegible]

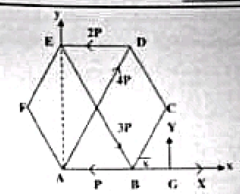
$$\frac{W(2 - \sqrt{3})}{\frac{W}{\sqrt{2}\cos 15}} = \sqrt{2}(2 - \sqrt{3})\cos 15 \le \mu \text{ [illegible]}$$

(μ [illegible])

(b) [illegible] x [illegible] y [illegible]

$$\rightarrow X = -P - 2P + 3P\cos 60 + 4P\cos 60$$

$$= \frac{P}{2}$$



$$\uparrow Y = 4P\cos 30 - 3P\cos 30$$

$$= \frac{P\sqrt{3}}{2}$$

$$R = \sqrt{X^2 + Y^2} = P$$

R [illegible] x [illegible] θ [illegible]

$$\tan\theta = \frac{Y}{X} = \sqrt{3} \Rightarrow \theta = 60^\circ$$

[illegible] AB [illegible] G [illegible] B [illegible] A [illegible] ($BG = \bar{x}$ [illegible])

$$2P \times 2a\cos 30 - 3P \times a\sin 60 = \frac{P\sqrt{3}}{2}(a + \bar{x})$$

$\bar{x} = 0$ [illegible]

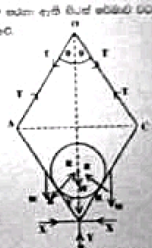
∴ [illegible] P, BC [illegible]

CB [illegible] P [illegible] FE [illegible] P [illegible] FE [illegible] BC [illegible] P [illegible] FE [illegible] P [illegible]

[illegible] AFEDCB [illegible] $P \times 2a\cos 30 = Pa\sqrt{3}$ වේ.

06. (a) [illegible] O [illegible]

∴ $Y = 0$



මුද්දේ සමතුලිතතාවය සලකා ↑ විභේදනයෙන්,

$2R\sin\theta = W \Rightarrow R = \frac{W}{2\sin\theta}$

පද්ධතියේ සමතුලිතතාවය සලකා ↑ විභේදනයෙන්,

$2T\cos\theta = 3W \Rightarrow T = \frac{3W}{2\cos\theta}$

AB දණ්ඩේ සමතුලිතතාවය සලකා, B වටා ඝූර්ණ ගැනීමෙන්,

$R\times\frac{a}{3}\cot\theta + W\times a\sin\theta = (T\cos\theta\times 2a\sin\theta) + (T\sin\theta\times 2a\cos\theta)$

$\frac{W}{2\sin\theta}\cdot\frac{a}{3}\frac{\cos\theta}{\sin\theta} + W\times a\sin\theta = 2\times\frac{3W}{2\cos\theta}\times 2a\sin\theta\cos\theta$

$\cos\theta + 6\sin^3\theta = 36\sin^3\theta$

$\frac{\cos\theta}{\sin^3\theta} = 30\,\frac{\sin^3\theta}{\sin^3\theta}$

$\cot\theta\,\mathrm{cosec}^2\theta = 30$

$\cot\theta\,(1+\cot^2\theta) = 30$

$\cot^3\theta + \cot\theta - 30 = 0$

$(\cot\theta - 3)(\cot^2\theta + 3\cot\theta + 10) = 0$

$\cot\theta = 3$ හෝ $\cot^2\theta + 3\cot\theta + 10 = 0$

මෙහි විවේචකය $\Delta = 9 - 40 < 0$

∴ θ සඳහා ඇත්තේ $\cot^{-1}3$ අගය පමණී.

AB සඳහා තිරස් විභේදනයෙන්,

$T\sin\theta + X = R\cos\theta$

$\frac{3W}{2\cos\theta}\sin\theta + X = \frac{W}{2\sin\theta}\times\cos\theta$

$X = \frac{W}{2}\cot\theta - \frac{3W}{2}\tan\theta$

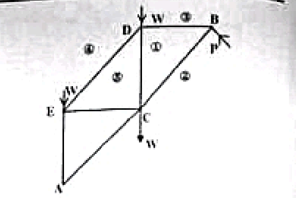
$= \frac{W}{2}\times 3 - \frac{3W}{2}\times\frac{1}{3} = W$

(b) සමාන දණ්ඩක දිග a යැයි සිතමු.
රාමු සැකිල්ලේ සමතුලිතතාව සලකා A වටා ඝූර්ණ ගැනීමෙන්,

$W\times a + W\times a = P\times 2a\sec 45^\circ$

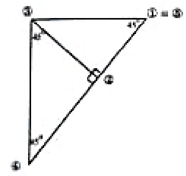
$2W = 2P\sqrt{2}$

$P = \frac{W}{\sqrt{2}}$

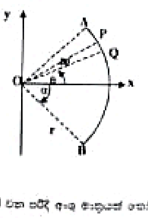


ප්‍රත්‍යාබල සටහන

දණ්ඩ	ප්‍රත්‍යාබලය	විශාලත්වය
BD	තෙරපුම	W
BC	ආතතිය	$W/\sqrt{2}$
DE	තෙරපුම	$\sqrt{2}\,W$



07. සමමිතිකත්වයෙන් චාපයේ ගුරුත්ව කේන්ද්‍රය G, x- අක්ෂය මත පිහිටයි.



$\hat{POQ} = \delta\theta$ වන පරිදි අංශු මාත්‍රයක් තෝරා ගනිමු

PQ චාප දිග $= r\delta\theta$

ρ_1 යනු ඒකක දිගක ස්කන්ධය නම්,

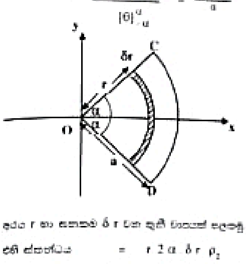
PQ චාපයේ ස්කන්ධය $= r\delta\theta\rho_1$

PQ චාප කොටසේ ස්කන්ධ කේන්ද්‍රයට y අක්ෂයේ සිට ඇති දුර $= r\cos\theta$

ස්කන්ධ කේන්ද්‍රයේ අර්ථ දැක්වීම අනුව,

$$\bar{x} = \frac{\int_{-\alpha}^{\alpha} r^2\cos\theta\, d\theta\,\rho_1}{\int_{-\alpha}^{\alpha} r\,d\theta\,\rho_1}$$

$$= \frac{r\,[\sin\theta]_{-\alpha}^{\alpha}}{[\theta]_{-\alpha}^{\alpha}} = \frac{r\sin\alpha}{\alpha}$$



අරය r හා ඝනකම δr වන තුනී චාපයක් සලකමු

එහි ස්කන්ධය $= r\,2\alpha\,\delta r\,\rho_2$

මෙහි ρ_2 යනු ආස්තරයේ ඒකක වර්ගඵලයක ස්කන්ධය වේ

මෙම තුනී චාපයේ ස්කන්ධ කේන්ද්‍රයට y අක්ෂයේ සිට ඇති දුර $= \frac{r\sin\alpha}{\alpha}$

∴ ආස්තරයේ ස්කන්ධ කේන්ද්‍රයට y අක්ෂයේ සිට ඇති දුර

$$\bar{x} = \frac{\int_0^a r\cdot 2\alpha\,dr\,\rho_2\times\frac{r\sin\alpha}{\alpha}}{\int_0^a r\cdot 2\alpha\,dr\,\rho_2}$$

$$= \frac{\sin\alpha}{\alpha}\left\{\frac{\left[\frac{r^3}{3}\right]_0^a}{\left[\frac{r^2}{2}\right]_0^a}\right\} = \frac{2}{3}\,\frac{a\sin\alpha}{\alpha}$$

OA $= a$

OC $= \frac{a}{\sqrt{3}}$

AC $= \frac{2a}{\sqrt{3}}$

ABE ◗ $= \frac{\pi a^2}{2}$

ABC △ $= \frac{1}{2}\times 2a\times\frac{a}{\sqrt{3}}$

ACBD◁ $= \frac{1}{2}\times\left(\frac{2a}{\sqrt{3}}\right)^2\times\frac{2\pi}{3}$

වස්තුව	ස්කන්ධය	ස්කන්ධ කේන්ද්‍රයට C සිට දුර
ABE අර්ධ වෘත්තාකාර ආස්තරය	$\frac{\pi a^2\rho}{2}$	$\frac{a}{\sqrt{3}} + \frac{4a}{3\pi}$
ABC ත්‍රිකෝණාකාර ආස්තරය	$\frac{a^2\rho}{\sqrt{3}}$	$\frac{2}{3}\cdot\frac{a}{\sqrt{3}}$
OADB කේන්ද්‍රික ඛණ්ඩය	$\frac{4\pi a^2}{9}\rho$	$\frac{2a}{\pi}$
ADBE ඉතිරි	$a^2\rho\left(\frac{\pi}{18}+\frac{1}{\sqrt{3}}\right)$	$\bar{x}$

C වටා ඝූර්ණ ගැනීමෙන්

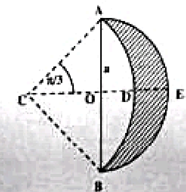
$$\frac{\pi a^2\rho}{2}\left(\frac{a}{\sqrt{3}}+\frac{4a}{3\pi}\right) + \frac{a^2\rho}{\sqrt{3}}\times\frac{2a}{3\sqrt{3}} - \frac{4\pi a^2\rho}{9}\times\frac{2a}{\pi} = a^2\rho\left(\frac{\pi}{18}+\frac{1}{\sqrt{3}}\right)\bar{x}$$

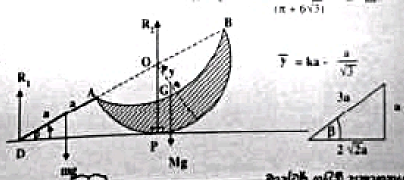
$$\frac{\pi a}{2\sqrt{3}} + \frac{2a}{3} + \frac{2a}{9} - \frac{8a}{9} = \left(\frac{\pi+6\sqrt{3}}{18}\right)\bar{x}$$

$$\frac{\pi a}{2\sqrt{3}}\times\frac{18}{(\pi+6\sqrt{3})} = \bar{x}$$

$$\therefore\ \bar{x} = \frac{3\sqrt{3}\,\pi\,a}{(\pi+6\sqrt{3})} = ka$$

07.





$\bar{y} = ka - \frac{a}{\sqrt{3}}$

[illegible] P [illegible]

$R_1 \times 3a\cos\beta + Mg\left(ka - \frac{a}{\sqrt{3}}\right)\sin\beta - mg \cdot 2a\cos\beta = 0$

[illegible] R_1 [illegible] D [illegible] D [illegible] $R_1 > 0$ [illegible]

[illegible] $mg \cdot 2\cancel{a}\cos\beta > Mg\left(k\cancel{a} - \frac{\cancel{a}}{\sqrt{3}}\right)\sin\beta$

$2m > M\frac{(\sqrt{3}k - 1)}{\sqrt{3}} \tan\beta$

$2m > M\frac{(\sqrt{3}k - 1)}{\sqrt{3}} \times \frac{1}{2\sqrt{2}}$

$4\sqrt{6}m > M(\sqrt{3}k - 1)$

08. B [illegible] A [illegible]

$P(B|A) = P\frac{(A \cap B)}{P(A)}$ [illegible] $P(A) > 0$ [illegible]

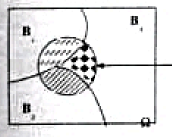
[illegible]

$P(C|A \cap B) = \frac{P(C \cap A \cap B)}{P(A \cap B)}$ [illegible]

$\therefore P(A) \cdot P(B|A)\, P(C|A \cap B) = P(A) \cdot \frac{P(B \cap A)}{P(A)} \cdot \frac{P(C \cap A \cap B)}{P(A \cap B)}$

$= P(A \cap B \cap C)$

[illegible]



$A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A)$

$P(A) = P[(B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A)]$

$= P(B_1 \cap A) + P(B_2 \cap A) + P(B_3 \cap A)$

$(\because$ [illegible] $B_1 \cap A,\ B_2 \cap A,\ B_3 \cap A$ [illegible]$)$

$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)$ ——— ①

([illegible])

$P(B_i|A) = \frac{P(A \cap B_i)}{P(A)} \quad i = 1, 2, 3$ [illegible]

$= \frac{P(B_i)P(A|B_i)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$

X_i : [illegible] $(i = 1, 2, 3)$ [illegible]

Y_i : [illegible] $(i = 1, 2, 3)$ [illegible]

Z_i : [illegible] $(i = 1, 2, 3)$ [illegible]

$P(X_i) = \frac{50}{100} = 0.5, \quad P(Y_i) = \frac{20}{100} = 0.2,$

$P(Z_i) = \frac{30}{100} = 0.3$

(i) [illegible]
$= Z_1 \cap Z_2 \cap Z_3$

[illegible] $= P(Z_1 \cap Z_2 \cap Z_3)$

$= P(Z_1)P(Z_2)P(Z_3)$

$(\because Z_1, Z_2, Z_3$ [illegible]$)$

$= 0.3 \times 0.3 \times 0.3 = 0.027$

(ii) [illegible]
$= (X_1 \cap X_2 \cap X_3) \cup (Y_1 \cap Y_2 \cap Y_3) \cup (Z_1 \cap Z_2 \cap Z_3)$

[illegible]
$= P[(X_1 \cap X_2 \cap X_3) \cup (Y_1 \cap Y_2 \cap Y_3) \cup (Z_1 \cap Z_2 \cap Z_3)]$

$= P(X_1 \cap X_2 \cap X_3) + P(Y_1 \cap Y_2 \cap Y_3) + P(Z_1 \cap Z_2 \cap Z_3)$

$(\because$ [illegible]$)$

$= P(X_1)P(X_2)P(X_3) + P(Y_1)P(Y_2)P(Y_3) + P(Z_1)P(Z_2)P(Z_3)$

$(\because$ [illegible]$)$

$(0.5)^3 + (0.2)^3 + (0.3)^3 = 0.125 + 0.008 + 0.027$

$0.160 = \underline{\underline{0.16}}$

(iii) [illegible]
$= (X_1 \cap Y_2 \cap Y_3) \cup (Y_1 \cap X_2 \cap Y_3) \cup (Y_1 \cap Y_2 \cap X_3)$

[illegible]
$= P(X_1 \cap Y_2 \cap Y_3) + P(Y_1 \cap X_2 \cap Y_3) + P(Y_1 \cap Y_2 \cap X_3)$

([illegible])

$= P(X_1)P(Y_2)P(Y_3) + P(Y_1)P(X_2)P(Y_3) + P(Y_1)P(Y_2)P(X_3)$

$= 0.5 \times 0.2 \times 0.2 + 0.2 \times 0.5 \times 0.2 + 0.2 \times 0.2 \times 0.5$

$= 3 \times 0.5 \times 0.2 \times 0.2$

$= 0.060$

(iv) [illegible]

$(X_1 \cap Y_2 \cap Z_3) \cup (Y_1 \cap Z_2 \cap X_3) \cup (Z_1 \cap X_2 \cap Y_3)$

[illegible] $= (0.5 \times 0.2 \times 0.3) + (0.2 \times 0.3 \times (0.5)) + (0.3 \times 0.5 \times 0.2)$

$= 3 \times 0.5 \times 0.2 \times 0.3$

$= \underline{\underline{0.090}}$

A : [illegible]

(a) [illegible]

[illegible] $= P[(X_1 \cap X_2 \cap X_3) | A]$

$= \frac{P(X_1 \cap X_2 \cap X_3 \cap A)}{P(A)}$

$= \frac{P(X_1 \cap X_2 \cap X_3)}{P(A)}$

$[\because X_1 \cap X_2 \cap X_3 \cap A = X_1 \cap X_2 \cap X_3$ [illegible]$]$

$= \frac{0.125}{0.16} = \frac{125}{160}$

$= \frac{25}{32} = 0.78125$

(b) [illegible]

$= \frac{0.008}{0.16} = \frac{1}{20}$

$= 0.05$

(c) [illegible]

$= \frac{0.027}{0.16} = \frac{27}{160}$

$= 0.16875$

[illegible]

09. (a) $\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} (x_i^2 - 2\bar{x}\, x_i + \bar{x}^2)$

$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2$

$= \sum_{i=1}^{n} x_i^2 - 2\bar{x} \cdot n\bar{x} + n\bar{x}^2$

$\left(\because \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i\right)$

$= \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

[illegible] 200

[illegible] x_i [illegible] $\quad \sum x_i = 920$

$\sum x_i^2 = 5032$

$n = 200$ [illegible]

[illegible] $\quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$= \frac{1}{200} \times 920$

$= \underline{4.6}$

[illegible] $\quad = \frac{1}{n}\sum_{i=1}^{200} x_i^2 - \bar{x}^2$

$= \frac{1}{200} \times 5032 - (4.6)^2$

$= 25.16 - 21.16 = 4$

$\therefore$ [illegible] $= \underline{\underline{2}}$

[illegible] 50

[illegible] y_i [illegible] $\quad \bar{y} = 4.4$

[illegible] $= 2.2$

$\therefore$ [illegible] $= 4.84$

$\sum_{i=1}^{n} y_i = n\bar{y}$

$= 50 \times 4.4$

$= 220$

[illegible] $4.84 = \frac{1}{50}\sum_{i=1}^{50} y_i^2 - \bar{y}^2$

$= \frac{1}{50}\sum_{i=1}^{50} y_i^2 - (4.4)^2$

$4.84 \times 50 = \sum_{i=1}^{50} y_i^2 - 19.36 \times 50$

$$\sum_{i=1}^{50} y_i^2 = 24.2 \times 50$$
$$= 1210$$

මුළු 250 සැලකූ විට, දෙවන ශ්‍රේණිය z_i නම්.

$$\sum_{i=1}^{250} z_i = 920 + 220$$
$$= 1140$$
$$\therefore \quad \bar{z} = \frac{1140}{250}$$
$$= \underline{\underline{4.56}}$$

විචලතාව s

$$= \frac{1}{250} \sum_{i=1}^{n} z_i^2 - \bar{z}^2$$
$$= \frac{1}{250}[5032 + 1210] - (4.56)^2$$
$$= 24.968 - 20.7936 = 4.1744$$

$\therefore$ සම්මත අපගමනය $= \sqrt{4.1744} \approx \underline{\underline{2.04}}$

(b) මුල් ලකුණ x ද, පරිවර්තිත ලකුණ y ද යැයි ගනිමු.
a හා b නියත විට, රේඛීය පරිණාමය

$y = a + bx$ —— ① යැයි සිතමු.

එවිට, $\bar{y} = a + b\bar{x}$ —— ② වේ

$\bar{x} = 45$ ද, $\bar{y} = 50$ ද, $x = 70$ ද $y = 80$

ද බව දී ඇත.

② න් $50 = a + 45b$ —— ③

① න් $80 = a + 70b$ —— ④

③ හා ④ විසඳීමෙන් $a = -4$ හා $b = 1.2$ බව ලැබේ.

(i) $\therefore$ රේඛීය පරිණාමය $y = -4 + 1.2x$ වේ.

(ii) විචලතාවයන් අතර සම්බන්ධය සඳහා.

$$\text{Var } x = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2 \text{ නිසා}$$
$$\text{Var } y = \frac{1}{n}\sum_{i=1}^{n} (y_i - \bar{y})^2$$
$$= \frac{1}{n}\sum_{i=1}^{n} [a + bx_i - (a + b\bar{x})]^2$$
$$= \frac{1}{n}\sum_{i=1}^{n} b^2(x_i - \bar{x})^2$$
$$= b^2 \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

$\therefore$ සම්මත අපගමනය $y = bx$ (සම්මත අපගමනය x)

සම්මත අපගමනය $y = 15$ වන විට හා $b = 1.2$ වන විට,

$$15 = 1.2 \times (\text{ස.අ.x})$$

$\therefore$ මුල් ලකුණුවල (x) සම්මත අපගමනය

$$= \frac{15}{1.2} = 12.5$$

(iii) පරිණාමනය මගින් ලකුණ වෙනස් නොවේ නම් $x = y$ විය යුතු ය.

$\therefore$ ① න්
$$x = a + bx$$
$$x = -4 + 1.2x$$
$$4 = 0.2x$$
$$\underline{\underline{x = 20}}$$

පරිණාමනය කරන ලද අවම ලකුණ 2 නම්
① න්

$2 = -4 + 1.2\, x_{min} \Rightarrow x_{min} = 5$

උපරිම ලකුණ 92 නම්.

$92 = -4 + 1.2_{max} \Rightarrow x_{max} = 80$

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