

## නව නිර්දේශය/புதிய பாடத்திட்டம்/New Syllabus

**NEW**

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව  
 இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம்  
 Department of Examinations, Sri Lanka  
 இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரīட்சைத் திணைக்களம்

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු  
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ட்  
 General Certificate of Education (Adv. Level) Examination, August 2019

ගණිතය I  
 கணிதம் I  
 Mathematics I

07 E I

28.08.2019 / 0830 - 1140

පැය තුනයි  
 மூன்று மணித்தியாலம்  
 Three hours

අමතර කියවීමේ කාලය - මිනිත්තු 10 යි  
 மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்  
 Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number

## Instructions:

- \* This question paper consists of two parts;  
**Part A** (Questions 1–10) and **Part B** (Questions 11–17).
- \* **Part A:**  
 Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- \* **Part B:**  
 Answer five questions only. Write your answers on the sheets provided.
- \* At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- \* You are permitted to remove **only Part B** of the question paper from the Examination Hall.

For Examiners' Use only

(07) Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	<b>Total</b>	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner	
Checked by:	1
	2
Supervised by:	





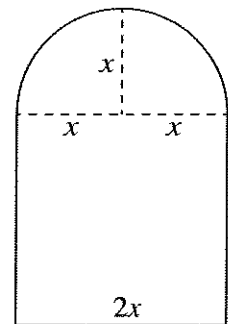








- 13.(a) Let  $a \in \mathbb{R}$ . Show that the roots of the equation  $x^2 + ax - 1 = 0$  are real and distinct.  
Let  $\alpha$  and  $\beta$  be these roots. Find the quadratic equation that has  $2\alpha + 1$  and  $2\beta + 1$  as its roots.
- (b) Let  $f(x) = x^3 + 3x^2 + px + q$ , where  $p$  and  $q$  are real numbers.  
The remainder when  $f(x)$  is divided by  $(x-1)$  is  $-12$  and  $(x-2)$  is a factor of  $f(x)$ . Find the values of  $p$  and  $q$ .  
Also, find the other linear factors of  $f(x)$ .
- 14.(a) Let  $k \in \mathbb{R}$ . The coefficients of  $x^{20}$  and  $x^{21}$  in the binomial expansion of  $(1 + kx)^{23}$  are equal. Show that  $k = 7$ .
- (b) By discarding the terms involving powers of  $x$  greater than 3, find an approximate value for  $(1.7)^{23} + (0.3)^{23}$ .
- (c) A person opened a bank account by depositing Rs.50000 at the beginning of a month. He then deposited Rs.20000 at the beginning of every month for two years. The account pays 0.5% interest compounded monthly. Find the balance in the account after two years.  
At the end of every month after this two year period, he withdrew Rs. 20000 from the account. For how long will there be money left in the account for him to continuously withdraw Rs.20000 per month?
15. Show that there are two straight lines  $l_1$  and  $l_2$  passing through the point  $(-2, 8)$  and sum of the intercepts on the axes is 6.  
A straight line meets the above two straight lines  $l_1$  and  $l_2$  at points  $P$  and  $Q$  respectively. If the midpoint of the line segment  $PQ$  is  $(1, 5)$ , find the equation of the line  $PQ$ .  
Show that the equation of the straight line passing through the point of intersection of straight lines  $l_1$  and  $l_2$ , and perpendicular to  $PQ$  is  $4y = x + 34$ .
- 16.(a) Find  $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^3 - a^3}$ .
- (b) Differentiate each of the following with respect to  $x$ :
- (i)  $\ln(x + e^{\sqrt{x}})$       (ii)  $(x + \sqrt{x^2 + a^2})^3$       (iii)  $\sqrt{\frac{1+e^x}{1-e^x}}$
- (c) A window is in the shape of a rectangle surmounted by a semicircle as shown in the figure. The entire perimeter of the window is  $(\pi + 4)$  m. By taking  $x$  m as the radius of the semicircle, show that the area of the window  $A \text{ m}^2$  is given by  $A = k(2x - x^2)$ , where  $k = \frac{1}{2}(\pi + 4)$ .  
Find the value of  $x$  such that the area of the window is maximum.



17.(a) Using the method of **integration by parts**, evaluate  $\int (x+1)^2 e^x dx$ .

(b) The following table gives the values of the function  $f(x) = \frac{1}{(2-x)^2}$  correct to four decimal places, for values of  $x$  between 0 and 1 at intervals of length 0.2.

$x$	0.00	0.20	0.40	0.60	0.80	1.00
$f(x)$	0.2500	0.3086	0.3906	0.5102	0.6944	1.0000

Using **Trapezoidal rule**, find an approximate value for  $I = \int_0^1 \frac{1}{(2-x)^2} dx$ , correct to three decimal places.

Find  $I$  using the substitution  $u=2-x$  or otherwise and compare with the approximation obtained above.

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9. Suppose that  $A$  and  $B$  are exhaustive events of a sample space  $S$ . If  $P(A) = \frac{2}{3}$  and  $P(A \cap B) = \frac{1}{5}$ , find (i)  $P(B)$ , (ii)  $P(A|B)$ , (iii)  $P(A'|B')$ .

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10. Let  $X$  be a continuous random variable with probability density function  $f(x)$  given by

$$f(x) = \begin{cases} k(3x-1), & 1 \leq x \leq 4, \\ 0, & \text{otherwise,} \end{cases}$$

where  $k$  is a positive constant.

Find

- (i) the value of  $k$ ,
- (ii) the mean of  $X$ .

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නව නිර්දේශය/புதிய பாடத்திட்டம் / New Syllabus


 ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව  
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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු  
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ඔගස්තු  
 General Certificate of Education (Adv. Level) Examination, August 2019

ගණිතය II  
 கணிதம் II  
**Mathematics II**

07 E II

**Part B**

\* Answer five questions only.

- II. A company owns two machines A and B which have different production capacities for high, medium and low grade nails. This company should produce at least 7, 6 and 13 tons per week of high, medium and low grade nails respectively to meet the demand of the market. It costs the company Rs. 10000 and Rs. 8000 per day to operate machines A and B respectively.

The following table gives the capacity of production in tons per day of each machine on each grade of nails.

Grade of nails	Capacity (tons/day)	
	A	B
High	2	1
Medium	1	1
Low	2	3

The company wishes to find out the number of days that each machine be operated per week to minimize the total production cost, while meeting the demand.

- Formulate this as a linear programming problem.
- Sketch the feasible region.
- Using the graphical method, find the solution of the problem formulated in part (i) above.
- Due to a technical issue, the machine B has to be operated for a week at most twice as many days as the machine A is operated.

Find the increase of total production cost per week, if the company still wishes to minimize the production cost.

12.(a) Let  $\mathbf{A} = \frac{1}{3} \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{pmatrix}$ .

Find  $x$  and  $y$  such that  $\mathbf{A}\mathbf{A}^T = \mathbf{I}_3$ , where  $\mathbf{I}_3$  is the identity matrix of order 3 and  $\mathbf{A}^T$  represents the transpose of  $\mathbf{A}$ .

(b) Let  $\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ .

Find constants  $p$  and  $q$  such that  $\mathbf{A}^3 + p\mathbf{A} = q\mathbf{I}_3$ , where  $\mathbf{I}_3$  is the identity matrix of order 3.

**Deduce** that there is a square matrix  $\mathbf{B}$  of order 3 such that  $\mathbf{B}\mathbf{A} = \mathbf{I}_3$ .

Consider the following system of linear equations:

$$\begin{aligned} y + z &= 1 \\ x + z &= 2 \\ x + y &= 5 \end{aligned}$$

By taking  $\mathbf{H} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$  and  $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ , show that the matrix equation  $\mathbf{A}\mathbf{X} = \mathbf{H}$  represents the above system of linear equations.

**Hence** solve the above system of linear equations.

13.(a) Two unbiased standard six-sided dice I and II with faces marked 1, 2, 3, 4, 5, 6 are tossed. Let  $x$  and  $y$  be the numbers landed in die I and die II respectively.

Let  $A$  and  $B$  be the events defined by

$$A : x \leq y, \text{ and}$$

$$B : x + y \text{ is an odd integer.}$$

Find  $P(A)$ ,  $P(B)$ ,  $P(A \cap B)$  and  $P(A|B)$ .

(b) (i) Find the number of different permutations that can be formed from the ten letters of the word "STATISTICS".

(ii) Find the number of different combinations that can be formed from four letters taken from the ten letters of the word "STATISTICS".

14. Fruits are packed in three boxes,  $A$ ,  $B$  and  $C$ , so that box  $A$  contains only 7 mangoes, box  $B$  contains 4 mangoes and 3 pears and box  $C$  contains 5 apples and 2 pears. Suppose that a box is selected at random and 2 fruits are randomly picked one after the other without replacement from the selected box.

Assuming that the selection of each box is equally likely, find the probability that

- both fruits selected are mangoes,
  - at least one of the selected fruits is a mango,
  - both fruits selected are mangoes given that one is a mango,
  - fruits are of different kinds.
15. A continuous random variable  $X$  has an exponential distribution with probability density function  $f(x)$  given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x > 0, \\ 0 & , \text{otherwise,} \end{cases}$$

where  $\lambda (> 0)$  is a parameter.

Find the mean and the variance of  $X$ .

The lifetime  $X$  of an electric equipment is exponentially distributed with a mean of 2 years. Find the cumulative distribution function of  $X$  and **hence** find the median of  $X$ .

(You may take  $e^{-0.7} \simeq 0.5$ .)

An equipment is randomly selected. Find the probability that

- the life time of the equipment will exceed  $1\frac{1}{2}$  years,
  - the equipment will fail before 2 years, given that the equipment had lasted more than  $1\frac{1}{2}$  years.  
(You need not simplify the answers.)
16. The mean and the standard deviation of the set of values  $\{x_i : i = 1, 2, \dots, n\}$  are  $\mu$  and  $\sigma$  respectively. Find the mean and the standard deviation of the set of values  $\{ax_i + b : i = 1, 2, \dots, n\}$ , where  $a$  and  $b$  are constants.

The following table summarises the ages (recorded to the nearest year) at the initial diagnosis of high blood sugar of a group of 70 diabetic patients.

Age	Number of patients
10 – 20	9
20 – 30	12
30 – 40	32
40 – 50	14
50 – 60	3

- Using a suitable linear transformation or otherwise, calculate the mean and the standard deviation of the given frequency distribution.
- Find the inter-quartile range of the above distribution.
- Two more patients who were both initially diagnosed with high blood sugar at the age of 55 joined the group. Find the inter-quartile range of the frequency distribution of the initial age of diagnosis of high blood sugar of all 72 patients.

17. Duration of activities in a project and the flow of activities are described in the following table:

Activity	Immediate predecessor(s)	Duration (in months)
A	–	2
B	A	2
C	A	3
D	B, C	4
E	B, D	5
F	–	8
G	E, F	1
H	E, G	2
I	H	4

- (i) Construct the project network.
- (ii) Prepare an activity schedule that includes earliest start time, earliest finish time, latest start time, latest finish time and float for each activity.
- (iii) What are the activities that cannot be delayed without extending the total duration of the project?
- (iv) Find the total duration of the project.
- (v) Due to external reasons, activity F is expected to take one more month than the regular duration. Determine whether the project can still be completed within the total duration calculated in part (iv) above.

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