AL/2021(2022)/07/E-I

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2021(2022)
General Certificate of Education (Adv. Level) Examination, 2021(2022)

ගණිතය I கணிதம் I **Mathematics I**	**07 E I**

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number

Instructions:

- *This question paper consists of two parts;* **Part A** (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
- **Part A:** *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
- *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(07) Mathematics I		
Part	**Question No.**	**Marks**
A	**1**	
	2	
	3	
	4	
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B	**11**	
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In Numbers	
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Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Let $A = \{x \in \mathbb{R} : |x + 2| < 4\}$ and $B = \{x \in \mathbb{R} : |x + 1| \geq 5\}$. Find $A \cap B$, $A \cap B'$ and $A' \cup B$.

2. Let A and B be subsets of a universal set S. Show that $A \cup (A \cup B')' = A \cup B$.
Represent the sets $A \cup B$ and $(A \cup B')'$ in two separate Venn diagrams.

[see page three

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3. Show that the compound proposition $(p \wedge \sim q) \Rightarrow r$ and the compound proposition $(\sim p \vee q) \vee r$ are logically equivalent.

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4. Let $m, n \in \mathbb{Z}$. Using the method of contradiction, prove that if $m(n^2 + 2n)$ is odd, then m and n are both odd.

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5. Using the change of base formula, show that $\log_{x^2} 4 = \frac{1}{2}\log_x 4$.

Hence, solve the equation $\log_x 4 + \log_{x^2} 4 = 3$ for x.

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6. Find all real values of x satisfying the inequality $\frac{x-6}{2-x} \leq x$.

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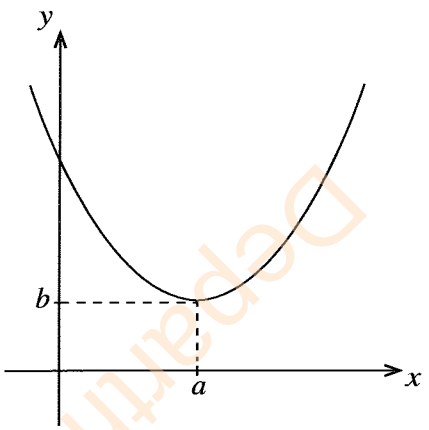
7. The graph of $f(x) = 2(x-2)^2 + 3$ is shown in the diagram. Write down the values of a and b, and the range of f.

State the largest value of k for which f has an inverse function on $x \leq k$.

For this value of k, find $f^{-1}(x)$.



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8. Let $A \equiv (3, 6)$ and $B \equiv (-5, 2)$. Find the equation of the perpendicular bisector l of AB.

Find the x–coordinates of the points on l such that the distance from the origin is 1 unit.

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9. A spherical balloon is expanding. At time t seconds, its radius is r cm. Its volume increases at a constant rate of $2\,\text{cm}^3\,\text{s}^{-1}$. Show that $\frac{dr}{dt} = \frac{1}{2\pi r^2}$.

Show also that the surface area of the balloon increases at a rate of $0.5\,\text{cm}^2\,\text{s}^{-1}$ when $r = 8$.

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10. Find the area of the region enclosed by the curves $y = x^4 - 1$ and $y = 1 - x^2$.

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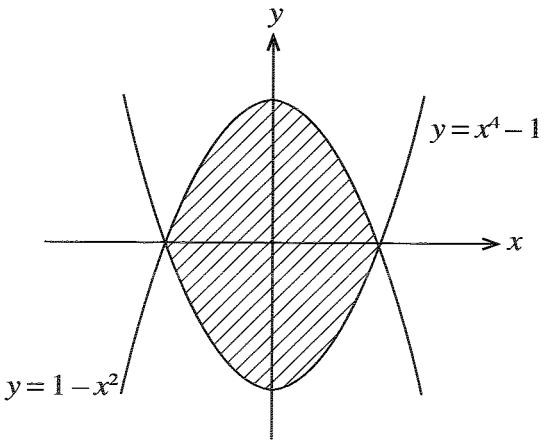

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General Certificate of Education (Adv. Level) Examination, 2021(2022)

ගණිතය I
கணிதம் I
Mathematics I

07 E I

Part B

* *Answer **five** questions only.*

11. (*a*) A player should pass two fitness tests to join a sports club. 120 players faced both fitness tests. It was found that the number of players who passed the first test is three times the number of players who passed both tests and, the number of players who passed the second test is twice the number of players who failed both tests. The number of players who passed only one test is 75.

Find the number of players

(i) who failed both tests.

(ii) who passed both tests.

(iii) who passed the first test.

(*b*) Using truth tables, determine whether each of the following compound propositions is a tautology or a contradiction.

(i) $\sim(p \to q) \vee (\sim p \vee (p \wedge q))$

(ii) $(p \to q) \wedge (q \to r) \wedge (p \wedge \sim r)$

12. (*a*) Using the **Principle of Mathematical Induction**, prove that

$$\sum_{r=1}^{n}\left(6r^2+1\right) = n\left(2n^2+3n+2\right) \text{ for all } n \in \mathbb{Z}^+.$$

(*b*) Let $U_r = \dfrac{3}{(3r-1)(3r+2)}$ for $r \in \mathbb{Z}^+$.

Verify that $U_r = \dfrac{1}{3r-1} - \dfrac{1}{3r+2}$ for $r \in \mathbb{Z}^+$.

Show that $\sum_{r=1}^{n} U_r = \dfrac{1}{2} - \dfrac{1}{3n+2}$ for $n \in \mathbb{Z}^+$.

Hence, show that $\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Deduce that $\sum_{r=1}^{\infty} U_{r+1} = \dfrac{1}{5}$.

[*See page eight*

13. (*a*) Let $k\left(\neq -\frac{1}{2}\right)$ be a real constant.

Show that the quadratic equation $(2k+1)x^2 - 2x - k = 0$ has real distinct roots.

Let $p = 2\alpha + \beta$ and $q = \alpha + 2\beta$, where α and β are the roots of the above equation. Express $p+q$ and pq in terms of k and find the quadratic equation whose roots are p and q.

(*b*) Let $p(x) = x^4 + 5x + a$, where a is a real constant.

If $p(x)$ is divisible by $x^2 - x + 3$, find the value of a and factorize $p(x)$ completely.

Hence, find all real roots of the equation $p(x) = 0$.

14. (*a*) Let $k \in \mathbb{R}$. Find in terms of k, the first 4 terms in the expansion of $(k+x)^8$ in ascending powers of x.

Given that the coefficients of x^2 and x^3 in this expansion are the same, find the value of k.

(*b*) A company earned a profit of Rs. 20 000 000 in the year 2020. The company considered two plans, Plan *A* and Plan *B* to increase profits. Under Plan *A*, the annual profit should increase each year by 5% of its value in the previous year. Under this plan find the total profit for the 10 years from 2020 to 2029.

Under Plan *B*, the annual profit should increase each year by a constant amount Rs. *D*. Find the value of *D* for which the total profit in the 10 years from 2020 to 2029 would be the same for both plans.

15. Let $A \equiv (1, a)$, $B \equiv (-3, b)$ and $M \equiv (c, 1)$, where $a, b, c \in \mathbb{R}$ and M is the mid-point of AB.

Find the value of c and show that the point $C \equiv (a-2, b-1)$ lies on the line l: $x + y + 1 = 0$.

It is given that AB is parallel to l.

Find the values of a and b.

With the above values for a, b and c, find

(i) the coordinates of the point D such that $ABCD$ is a parallelogram,

(ii) the area of the parallelogram $ABCD$.

Let m be the line $2x + y = 3$. Find the equation of the line through the point of intersection of l and m, and perpendicular to BD.

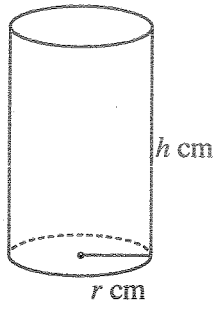
16. (*a*) Evaluate $\lim\limits_{x \to 2} \dfrac{(x^2-4)^3}{(x-2)} \cdot \dfrac{2}{\left(\sqrt{x} - \sqrt{2}\right)^2}$.

(*b*) Differentiate each of the following with respect to x:

(i) $\dfrac{3x^2+1}{x^2+3}$ (ii) $x^8 \ln x + \dfrac{(x+1)}{\ln x}$ (iii) $\sqrt{(e^{2x}+1)^2 + 1}$

(*c*) A closed cylindrical can that has a volume of 128π cm^3 needs to be constructed. Let its radius be r cm and the height be h cm as shown in the figure. Show that the total surface area S cm^2 of the can is given by $S = 2\pi\left(r^2 + \dfrac{128}{r}\right)$ for $r > 0$.

Find the value of r that minimizes S.



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17. (*a*) Using the method of **partial fractions**, find $\int \frac{1}{(x-1)(x-2)^2}\,dx$.

(*b*) Using the method of **integration by parts**, find $\int x\left(e^x + 2e^{2x}\right)dx$.

(*c*) The following table gives the values of the function $f(x) = xe^{x^2}$, correct to three decimal places, for values of x between 0 and 1 at intervals of length 0.25.

x	0	0.25	0.5	0.75	1
$f(x)$	0	0.266	0.642	1.316	2.718

Using **Simpson's rule**, find an approximate value for $I = \int_0^1 xe^{x^2}\,dx$.

Hence, find an approximate value for e.

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General Certificate of Education (Adv. Level) Examination, 2021(2022)

ගණිතය II கணிதம் II **Mathematics II**	**07 E II**

පැය තුනයි மூன்று மணித்தியாலம் ***Three hours***	අමතර කියවීම් කාලය - මිනිත්තු 10 යි மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள் **Additional Reading Time - 10 minutes**

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

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* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*
* *Statistical tables will be provided.*

For Examiners' Use only

(07) Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
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	5	
	6	
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B	11	
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Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Show that

$$\begin{vmatrix} a^2 & b^2 & b^2+ab \\ a^2+ab & b^2 & ab \\ ab & 2b^2 & b^2 \end{vmatrix} = 4a^2b^4, \text{ where } a, b \in \mathbb{R}.$$

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2. Let $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}$. Find **AB** and **A(2B – C)**.

Verify that **2AB – AC = A(2B – C)**.

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3. Daily rainfall measurements of 5 days were observed. The sum and the sum of squares of these measurements are 45 ml and 650 ml^2 respectively. Find the mean and standard deviation of the rainfall measurements.
The daily rainfall measurements of the next two days were also observed and those values are 10 ml and 8 ml. Find the new value of the mean.

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4. A book that has 200 pages contains 20 printing mistakes and these mistakes are randomly distributed. The number of mistakes per page has a Poisson distribution. Find the probability that 10 pages selected at random have only one mistake.

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5. The monthly salaries, in thousand rupees, of employees of a company are normally distributed with a mean of 80 and a standard deviation of 25. The company provides free transport to the employees who are in the lowest 10% of the salaries. Find the highest salary earned by the employees who are eligible for free transport.

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6. A survey report states that 15% of a certain type of remote controller stops working within the first year of its manufacturing. If 5 remote controllers of that type are selected at random, find the probability that

(i) 3 of them stop working within the first year,

(ii) more than 2 of them stop working within the first year.

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7. The probabilities that a person gets the first and the second job he applied for are 0.5 and 0.3 respectively. The probability that he gets both jobs he applied for is 0.4. Find the probability,

(i) that he gets at least one of the jobs he applied for,

(ii) that he gets the second job he applied for, provided that he got the first job he applied for.

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8. Let A and B be two events of a sample space S such that $P(A)=\frac{1}{4}$, $P(A'\cap B)=\frac{1}{2}$ and $P(B\,|\,A)=\frac{4}{5}$. Find (i) $P(A\cup B)$, (ii) $P(A\cap B)$ and (iii) $P(B)$.

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[see page six

9. According to past records, the probability distribution of the number of students X who are late for a class is given below:

k	0	1	2	3	4
$P(X = k)$	p	0.3	$3p$	0.2	p

Find the value of the constant p and show that $E(X) = 1.9$.

Let Y be the random variable given by $2X + 3$. Find $E(Y)$.

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10. A continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} 2ax - 3bx^2 & , \text{ for } 0 \le x \le 1, \\ 0 & , \text{ otherwise,} \end{cases}$$

where a and b are constants.

It is given that $E(X^2) = \frac{1}{4}$. Find the values of a and b.

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General Certificate of Education (Adv. Level) Examination, 2021(2022)

ගණිතය II
கணிதம் II
Mathematics II

07 E II

Part B

* *Answer **five** questions only.*

11. A supermarket makes three types of hampers: basic, standard and luxury.

Every basic hamper has 6 packets, 9 bottles and 6 tins.

Every standard hamper has 9 packets, 6 bottles and 8 tins.

Every luxury hamper has 9 packets, 9 bottles and 10 tins.

Each day, the supermarket must use at least 720 packets and at least 720 bottles whilst the maximum number of tins that can be used is at most 900.

On a particular day, the supermarket packs an equal number of standard hampers and luxury hampers.

If the supermarket makes a profit of Rs. 100, Rs. 200 and Rs. 500 from each basic, standard and luxury hamper and wishes to maximize the total profit.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Using the graphical method, find the solution of the problem formulated in part (i) above.

12. (*a*) Let $\mathbf{A} = \begin{pmatrix} a & 0 & 2 \\ 2 & b & 0 \end{pmatrix}$.

Find $\mathbf{AA^T}$ in terms of a and b.

Show that if $\mathbf{AA^T} = \begin{pmatrix} 5 & 2 \\ 2 & 8 \end{pmatrix}$, then $a = 1$ and $b = 2$.

Let $\mathbf{C} = \mathbf{AA^T} - 8\mathbf{I}$. Find $\mathbf{C^{-1}}$.

Find the matrix $\mathbf{D}$ such that $\mathbf{CD} = 8\mathbf{C} + \mathbf{I}$, where $\mathbf{I}$ is the identity matrix of order 2.

[*See page eight*

(*b*) Let $a, b \in \mathbb{R}$. Write the pair of simultaneous equations

$$ax + (b-1)y = 2$$
$$x - y = -4$$

in the form $\mathbf{PX} = \mathbf{Q}$, where $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$, and $\mathbf{P}$ and $\mathbf{Q}$ are matrices to be determined.

It is given that $\mathbf{X} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ is a solution of $\mathbf{PX} = \mathbf{Q}$. Show that $b = a + 2$.

Show that the above pair of equations has

(i) a unique solution when $a \neq -\frac{1}{2}$

(ii) infinitely many solutions when $a = -\frac{1}{2}$

13. (*a*) Two unbiased coins and an unbiased cubic die were tossed. Let A be the event of getting heads on both coins and B be the event of getting an even number on the die.
Find $P(A)$, $P(B)$ and $P(A \cup B)$.

(*b*) How many 6 digit telephone numbers can be formed such that the first and second digits are 3 and 5 respectively and no digit is repeated?
How many of these telephone numbers end with an odd digit?

(*c*) There are 8 men and 10 women in a group. In how many ways can a committee be formed from this group, consisting of

(i) 5 men and 6 women?

(ii) 6 members with at least 3 men?

14. A box contains 3 green balls and 2 blue balls, which are identical except for colour. A ball is drawn at random from the box. If the ball drawn is green, 2 other blue balls are added to the box without replacing the ball drawn and, if the ball drawn is blue, 2 other green balls are added to the box without replacing the ball drawn. Now, a second ball is drawn at random. Find the probability that

(i) both balls drawn are green,

(ii) at least one of the balls drawn is green,

(iii) both balls drawn are green, given that one of the balls drawn is green,

(iv) the balls drawn are of different colours.

[*See page nine*

15. A continuous random variable Y follows an exponential distribution with probability density function $f(y)$ given as follows:

$$f(y)=\begin{cases}\lambda e^{-\lambda y} & , \quad y>0 \\ 0 & , \quad \text{otherwise}\end{cases}$$

where the parameter $\lambda > 0$.

Find the mean, variance and the cumulative density function of Y.

Let the time taken by a doctor to treat a patient be exponentially distributed with a mean of 10 minutes. Find each of the following. (Answers need not be simplified.)

(i) The 50^{th} percentile of the time taken by the doctor to treat a patient.

(ii) The probability that the doctor takes more than 8 minutes to treat a patient.

(iii) If the doctor has already spent more than 10 minutes to treat a patient, the probability that he will finish treating this patient in less than 15 minutes.

16. (*a*) The following table gives the frequency distribution of the number of students who were absent for a class within a month.

No. of absent students	No. of days
1 – 3	15
4 – 6	12
7 – 9	10
10 – 12	5
13 – 15	2

Estimate the mean, mode and median of the distribution.

(*b*) The time spent by a barber to cut a customer's hair is normally distributed with a mean 20 minutes and a standard deviation of 5 minutes.

(i) Find the probability that the barber takes

(*a*) more than 25 minutes,

(*b*) a time between 25 and 30 minutes,

to cut a customer's hair.

(ii) Find the probability that he serves 5 customers in less than 2 hours (120 minutes).

[*See page ten*

17. The duration of activities of a project and the flow of activities are given in the following table:

Activity	Preceding Activity (Activities)	Duration (in months)
A	–	3
B	A	6
C	A	7
D	A	5
E	B, C	13
F	C, D	8
G	D, F	11
H	G, E	6
I	H	2

(i) Construct the project network.

(ii) Prepare an activity schedule that indicates earliest start time, earliest finish time, latest start time, latest finish time and float for each activity.

(iii) Find the total duration of the project.

(iv) Write down the critical path of the project.

(v) What are the activities that can be delayed without extending the duration of the project?

(vi) How is the project completion time affected by each of the following?

(*a*) Activity F is delayed by 2 months,

(*b*) Activity E is delayed by 1 month.

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