

සියලු ම හිමිකම් ඇවිරිණි / முழுப் பதிப்புரிமையுடையது / All Rights Reserved]

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව  
 இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம்  
 Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka  
 இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரīட்சைத் திணைக்களம் இலங்கைப் பரīட்சைத் திணைக்களம்  
 Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024  
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024  
 General Certificate of Education (Adv. Level) Examination, 2024

ගණිතය I  
 கணிதம் I  
 Mathematics I

07 E I

පැය තුනයි  
 மூன்று மணித்தியாலம்  
 Three hours

අමතර කියවීමේ කාලය - මිනිත්තු 10 යි  
 மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்  
 Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number

### Instructions:

- \* This question paper consists of two parts;  
**Part A** (Questions 1–10) and **Part B** (Questions 11–17).
- \* **Part A:**  
 Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- \* **Part B:**  
 Answer **five** questions only. Write your answers on the sheets provided.
- \* At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- \* You are permitted to remove **only Part B** of the question paper from the Examination Hall.

### For Examiners' Use only

(07) Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner	
Checked by:	1
	2
Supervised by:	







7. The functions  $f$  and  $g$  are defined by  $f(x) = 3x + 2$  and  $g(x) = \frac{6}{2x+3}$ .  
Find the value of  $x$  for which  $f(g(x)) = 3$ .  
Solve the equation  $f^{-1}(g(x)) = 1$  for  $x$ .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8. Let  $A \equiv (5, b)$  and  $B \equiv (-1, 3b)$  be two points, where  $b \in \mathbb{R}$ . Also, let  $C$  be the mid-point of  $AB$ .  
Find the values of  $b$  such that  $AB$  is perpendicular to  $OC$ , where  $O$  is the origin.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Department of Examinations Sri Lanka







අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024  
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024  
 General Certificate of Education (Adv. Level) Examination, 2024

ගණිතය I  
 கணிதம் I  
 Mathematics I

07 E I

## Part B

\* Answer five questions only.

11.(a) While observing the G.C.E. (Ordinary Level) results for the subjects Mathematics, Science and English of 40 students in a G.C.E. (Advanced Level) class, the following information were revealed:

- 20 students for Mathematics and 18 students for Science have obtained A grades.
- 9 students for both Mathematics and Science, 8 students for both Science and English, and 5 students for both Mathematics and English have obtained A grades.
- 10 students have obtained A grades only for two of the subjects among these subjects.
- The number of students who have not obtained A grades for any of these three subjects is twice the number of students who have obtained A grades for all three subjects.

Find the number of students

- who have obtained A grades for all three subjects,
- who have obtained A grade only for Mathematics,
- who have obtained A grade for English.

(b) Using truth tables, show that

- $(p \wedge q) \vee r$  is logically equivalent to  $(p \vee r) \wedge (q \vee r)$ ,
- $(p \vee q) \wedge (\sim p \vee r) \Rightarrow q \vee r$  is a tautology.

12.(a) Using the Principle of Mathematical Induction, prove that

$$\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{(2n+1)} \text{ for all } n \in \mathbb{Z}^+.$$

(b) Let  $f(r) = \frac{r}{(r+2)^2}$  for  $r \in \mathbb{Z}^+$ .

Find  $U_r$  such that  $U_r = f(r) - f(r+1)$  for  $r \in \mathbb{Z}^+$ .

Show that  $\sum_{r=1}^n U_r = \frac{1}{9} - \frac{(n+1)}{(n+3)^2}$  for  $n \in \mathbb{Z}^+$ .

Hence, show that  $\sum_{r=1}^{\infty} U_r$  is convergent and find its sum.

Also, find  $\sum_{r=15}^{\infty} U_r$ .

13.(a) The roots of the equation  $x^2 + px + q = 0$  are  $-3$  and  $5$ ; where  $p, q \in \mathbb{R}$ .

Find the values of  $p$  and  $q$ .

Using these values of  $p$  and  $q$ , find the value of the constant  $r (\in \mathbb{R})$  for which the equation,  $x^2 + px + q + r = 0$  has equal roots.

(b) Let  $p(x) = 2x^3 + ax^2 + (a+4)x + 6$ , where  $a \in \mathbb{R}$ . It is given that  $(x-2)$  is a factor of  $p(x)$ .

Find the value of  $a$ .

When  $a$  has this value, factorize  $p(x)$ .

Also, solve the inequality  $p(x) > 0$ .

14.(a) Find the first 3 terms in the expansion of  $(1-x-x^2)^6$  in ascending powers of  $x$ .

**Hence**, find the value of  $k$  such that the coefficient of  $x^2$  in the expansion of  $(k+x^2)(1-x-x^2)^6$  is 10.

(b) A person takes a loan of Rs. 500 000 from a bank that charges a monthly interest of 1%. He pays back the loan in equal monthly instalments of Rs.  $B$  over 5 years. Let Rs.  $A_n$  be the amount to be paid back after  $n$  months. Show that

$$A_2 = 500\,000 (1.01)^2 - B(1 + 1.01) \text{ and}$$

$$A_3 = 500\,000 (1.01)^3 - B(1 + 1.01 + 1.01^2).$$

Write down a similar expression for  $A_n$  in terms of  $n (\leq 60)$  and  $B$ .

Find the value of  $B$ .

15. Let the line  $l_1$  be given by  $4y = 3x + 1$ . The point  $A \equiv (a, 4)$  lies on  $l_1$ . Find the value of  $a$ .

Suppose that the line  $l_2$  is parallel to  $l_1$  and passing through the point  $(4, -3)$ . Find the equation of  $l_2$ .

Let  $B$  be the point of intersection of  $l_2$  and the  $y$ -axis, and  $C$  be the point on  $l_1$  such that  $BC$  is perpendicular to  $l_1$ . Find the coordinates of  $C$ .

Let  $D$  be the point such that  $ABCD$  is a parallelogram. Find the coordinates of  $D$ .

Also, find the area of  $ABCD$ .

අධ්‍යයන පොදු ඝනකී පත්‍ර (උසස් පෙළ) විභාගය, 2024  
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024  
 General Certificate of Education (Adv. Level) Examination, 2024

ගණිතය I  
 கணிதம் I  
 Mathematics I

07 E I

Part B

\* Answer five questions only.

11. (a) While observing the G.C.E. (Ordinary Level) results for the subjects Mathematics, Science and English of 40 students in a G.C.E. (Advanced Level) class, the following information were revealed:

- 20 students for Mathematics and 18 students for Science have obtained A grades.
- 9 students for both Mathematics and Science, 8 students for both Science and English, and 5 students for both Mathematics and English have obtained A grades.
- 10 students have obtained A grades only for two of the subjects among these subjects.
- The number of students who have not obtained A grades for any of these three subjects is twice the number of students who have obtained A grades for all three subjects.

Find the number of students

- who have obtained A grades for all three subjects,
- who have obtained A grade only for Mathematics,
- who have obtained A grade for English.

(b) Using truth tables, show that

- $(p \wedge q) \vee r$  is logically equivalent to  $(p \vee r) \wedge (q \vee r)$ ,
- $(p \vee q) \wedge (\sim p \vee r) \Rightarrow q \vee r$  is a tautology.

12. (a) Using the Principle of Mathematical Induction, prove that

$$\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{(2n+1)} \text{ for all } n \in \mathbb{Z}^+.$$

(b) Let  $f(r) = \frac{r}{(r+2)^2}$  for  $r \in \mathbb{Z}^+$ .

Find  $U_r$  such that  $U_r = f(r) - f(r+1)$  for  $r \in \mathbb{Z}^+$ .

Show that  $\sum_{r=1}^n U_r = \frac{1}{9} - \frac{(n+1)}{(n+3)^2}$  for  $n \in \mathbb{Z}^+$ .

Hence, show that  $\sum_{r=1}^{\infty} U_r$  is convergent and find its sum.

Also, find  $\sum_{r=15}^{\infty} U_r$ .

13.(a) The roots of the equation  $x^2 + px + q = 0$  are  $-3$  and  $5$ ; where  $p, q \in \mathbb{R}$ .

Find the values of  $p$  and  $q$ .

Using these values of  $p$  and  $q$ , find the value of the constant  $r (\in \mathbb{R})$  for which the equation,  $x^2 + px + q + r = 0$  has equal roots.

(b) Let  $p(x) = 2x^3 + ax^2 + (a+4)x + 6$ , where  $a \in \mathbb{R}$ . It is given that  $(x-2)$  is a factor of  $p(x)$ .

Find the value of  $a$ .

When  $a$  has this value, factorize  $p(x)$ .

Also, solve the inequality  $p(x) > 0$ .

14.(a) Find the first 3 terms in the expansion of  $(1-x-x^2)^6$  in ascending powers of  $x$ .

**Hence**, find the value of  $k$  such that the coefficient of  $x^2$  in the expansion of  $(k+x^2)(1-x-x^2)^6$  is 10.

(b) A person takes a loan of Rs. 500 000 from a bank that charges a monthly interest of 1%. He pays back the loan in equal monthly instalments of Rs.  $B$  over 5 years. Let Rs.  $A_n$  be the amount to be paid back after  $n$  months. Show that

$$A_2 = 500\,000 (1.01)^2 - B(1 + 1.01) \text{ and}$$

$$A_3 = 500\,000 (1.01)^3 - B(1 + 1.01 + 1.01^2).$$

Write down a similar expression for  $A_n$  in terms of  $n (\leq 60)$  and  $B$ .

Find the value of  $B$ .

15. Let the line  $l_1$  be given by  $4y = 3x + 1$ . The point  $A \equiv (a, 4)$  lies on  $l_1$ . Find the value of  $a$ .

Suppose that the line  $l_2$  is parallel to  $l_1$  and passing through the point  $(4, -3)$ . Find the equation of  $l_2$ .

Let  $B$  be the point of intersection of  $l_2$  and the  $y$ -axis, and  $C$  be the point on  $l_1$  such that  $BC$  is perpendicular to  $l_1$ . Find the coordinates of  $C$ .

Let  $D$  be the point such that  $ABCD$  is a parallelogram. Find the coordinates of  $D$ .

Also, find the area of  $ABCD$ .

16.(a) Evaluate  $\lim_{x \rightarrow 1} \frac{(x^3 - 1)^2}{(\sqrt{x} - 1)(2x^2 - x - 1)}$ .

(b) Differentiate each of the following with respect to  $x$ :

(i)  $\sqrt{\ln(x^4 e^x + 5x^2 + 3)}$ , (ii)  $(2 - 3x^2)^7 (x + 2x^2)^5$ , (iii)  $\frac{3e^{x^2} + 2x^3}{3e^{x^2} - 2x^3}$ .

(c) A right circular hollow cylinder, open at one end, is constructed from a thin sheet of metal. The total outer surface area of the cylinder is  $192\pi \text{ cm}^2$ . The cylinder has a radius of  $r \text{ cm}$  and a height of  $h \text{ cm}$ .

Express  $h$  in terms of  $r$ , and show that the volume  $V \text{ cm}^3$ , of the cylinder is given by  $V = \frac{1}{2}\pi(192r - r^3)$ .

Find the value of  $r$  such that  $V$  is maximum.

17.(a) Using **partial fractions**, find the value of  $\int_1^2 \frac{1}{x^2(x+1)} dx$ .

(b) Using **integration by parts**, find the value of  $\int_1^2 (12x^3 + 4x) \ln x dx$ .

(c) The following table gives the values of the function  $f(x) = \ln(4 + x^3)$ , correct to three decimal places, for values of  $x$  between 1 and 2 at intervals of length 0.25:

$x$	1	1.25	1.5	1.75	2.0
$f(x)$	1.609	1.784	1.998	2.236	2.485

Using **Simpson Rule**, find an approximate value for  $\int_1^2 \ln(4 + x^3) dx$ .

**Hence**, find an approximate value for  $\int_1^2 \ln\left(\frac{1}{4 + x^3}\right) dx$ .

\*\*\*

Department of Examinations Sri Lanka

Department of Examinations Sri Lanka

16. (a) Evaluate  $\lim_{x \rightarrow 1} \frac{(x^3 - 1)^2}{\sqrt{x-1}(2x^2 - x - 1)}$ .

(b) Differentiate each of the following with respect to  $x$ :

(i)  $\sqrt{\ln(x^4 e^x + 5x^2 + 3)}$ , (ii)  $(2 - 3x^2)^7 (x + 2x^2)^5$ , (iii)  $\frac{3e^{x^2} + 2x^3}{3e^{x^2} - 2x^3}$ .

(c) A right circular hollow cylinder, open at one end, is constructed from a thin sheet of metal. The total outer surface area of the cylinder is  $192\pi \text{ cm}^2$ . The cylinder has a radius of  $r \text{ cm}$  and a height of  $h \text{ cm}$ .

Express  $h$  in terms of  $r$ , and show that the volume  $V \text{ cm}^3$  of the cylinder is given by  $V = \frac{2}{1}\pi(192r - r^3)$ .

Find the value of  $r$  such that  $V$  is maximum.

17. (a) Using partial fractions, find the value of  $\int_2^1 \frac{1}{x^2(x+1)} dx$ .

(b) Using integration by parts, find the value of  $\int_2^1 (12x^3 + 4x) \ln x dx$ .

(c) The following table gives the values of the function  $f(x) = \ln(4 + x^3)$ , correct to three decimal places, for values of  $x$  between 1 and 2 at intervals of length 0.25:

$x$	$f(x)$
1	1.609
1.25	1.784
1.5	1.998
1.75	2.236
2.0	2.485

Using Simpson Rule, find an approximate value for  $\int_2^1 \ln(4 + x^3) dx$ .

Hence, find an approximate value for  $\int_2^1 \ln\left(\frac{4 + x^3}{1}\right) dx$ .

\*\*\*

Department of Examinations Sri Lanka

Department of Examinations Sri Lanka

(a) Evaluate  $\lim_{x \rightarrow 1} \frac{(x^3 - 1)^2}{(\sqrt{x} - 1)(2x^2 - x - 1)}$ .

(b) Differentiate each of the following with respect to  $x$ :

(i)  $\sqrt{\ln(x^4 e^x + 5x^2 + 3)}$ , (ii)  $(2 - 3x^2)^7 (x + 2x^2)^5$ , (iii)  $\frac{3e^{x^2} + 2x^3}{3e^{x^2} - 2x^3}$ .

(c) A right circular hollow cylinder, open at one end, is constructed from a thin sheet of metal. The total outer surface area of the cylinder is  $192\pi \text{ cm}^2$ . The cylinder has a radius of  $r \text{ cm}$  and a height of  $h \text{ cm}$ .

Express  $h$  in terms of  $r$ , and show that the volume  $V \text{ cm}^3$  of the cylinder is given by  $V = \frac{2}{1}\pi(192r - r^3)$ .

Find the value of  $r$  such that  $V$  is maximum.

(a) Using partial fractions, find the value of  $\int_2^1 \frac{1}{x^2(x+1)} dx$ .

(b) Using integration by parts, find the value of  $\int_2^1 (12x^3 + 4x) \ln x \, dx$ .

(c) The following table gives the values of the function  $f(x) = \ln(4 + x^3)$ , correct to three decimal places, for values of  $x$  between 1 and 2 at intervals of length 0.25:

$x$	1	1.25	1.5	1.75	2.0
$f(x)$	1.609	1.784	1.998	2.236	2.485

Using Simpson Rule, find an approximate value for  $\int_2^1 \ln(4 + x^3) \, dx$ .

Hence, find an approximate value for  $\int_2^1 \ln\left(\frac{1}{4 + x^3}\right) \, dx$ .

\*\*\*

Department of Examinations Sri Lanka  
Department of Examinations Sri Lanka

මෙහි ඇති ප්‍රශ්නවලට පිළිතුරු ලබාදීම සඳහා ලබාදෙන කාලය ඔබගේ ප්‍රශ්නපත්‍රිකාවේ මුද්‍රණය කර ඇති කාලයට වඩා වැඩි නොවිය යුතුය. ඔබගේ ප්‍රශ්නපත්‍රිකාවේ මුද්‍රණය කර ඇති කාලයට වඩා වැඩි කාලයක් ලබාදීමට අපට හැකියාවක් නැත.

இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம்

Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka

இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரīட்சைத் திணைக்களம் இலங்கைப் பரīட்சைத் திணைக்களம்

අවසාන වෛද්‍ය සහතික වනු (සේස් වෙළ) විභාගය, 2024  
 கல்விப் பொதுப் பத்திர (உயர் தர) பரீட்சை, 2024  
 General Certificate of Education (Adv. Level) Examination, 2024

මෙහි ඇති ප්‍රශ්නවලට පිළිතුරු ලබාදීම සඳහා ලබාදෙන කාලය ඔබගේ ප්‍රශ්නපත්‍රිකාවේ මුද්‍රණය කර ඇති කාලයට වඩා වැඩි නොවිය යුතුය. ඔබගේ ප්‍රශ්නපත්‍රිකාවේ මුද්‍රණය කර ඇති කාලයට වඩා වැඩි කාලයක් ලබාදීමට අපට හැකියාවක් නැත.

07 E II

අමතර ක්‍රියාත්මක කාලය - මිනිත්තු 10 යි  
 අමතර ක්‍රියාත්මක කාලය - 10 நிமிடங்கள்  
 Additional Reading Time - 10 minutes

Three hours  
 மூன்று மணித்தியாலம்

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							
--------------	--	--	--	--	--	--	--

**Instructions:**

- \* This question paper consists of two parts: Part A (Questions 1–10) and Part B (Questions 11–17).
- Part A: Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- Part B: Answer five questions only. Write your answers on the sheets provided.
- \* At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on top of Part B and hand them over to the supervisor.
- \* You are permitted to remove only Part B of the question paper from the Examination Hall.
- \* Statistical tables will be provided.

For Examiners' Use only

Part	Question No.	Marks
A	1	2
	2	2
	3	3
	4	4
	5	5
	6	6
	7	7
	8	8
	9	9
	10	10
B	11	11
	12	12
	13	13
	14	14
	15	15
	16	16
	17	17
Total		

In Numbers	
In Words	
Total	
Marking Examiner	
Checked by:	1
Supervised by:	2
Code Numbers	

1. Let  $a \in \mathbb{R}$ .

$$\text{Show that } \begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (1-a^3)^2.$$

2. Let  $A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ . Find  $A^2$ .  
 Find the value of  $k \in \mathbb{R}$  such that  $A^2 = kA - 2I$ , where  $I$  is the  $2 \times 2$  identity matrix.



5. Heights of a group of persons are normally distributed with a mean of 58 inches and a standard deviation of 4 inches. Find the probability that the height of a randomly selected person from this group is
- (i) at most 60 inches,
  - (ii) at least 56 inches, given that the person is shorter than 60 inches.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6. The cards in a pack of five cards are labelled as 1, 2, 3, 4 and 5. Cards are randomly drawn from the pack, with replacement, until an even numbered card is obtained. Find the probability that an even numbered card is obtained
- (i) at the first draw,
  - (ii) before the third draw,
  - (iii) at the third draw.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Department of Examinations Sri Lanka









ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව  
 இலங்கைப் பரீட்சைத் திணைக்களம்  
 Department of Examinations, Sri Lanka  
 இலங்கைப் பரීட்சைத் திணைக்களம்  
 Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2024  
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2024  
 General Certificate of Education (Adv. Level) Examination, 2024

ගණිතය II  
 கணிதம் II  
 Mathematics II

07 E II

## Part B

\* Answer five questions only.

11. A manufacturer produces a certain product in two qualities: Grade I and Grade II. The same raw materials are used to produce both types. The amount of raw materials and labour hours needed to produce one unit in each quality and the profit per unit in each quality are shown in the table below:

	Per unit values	
	Grade I product	Grade II product
Raw materials (kg)	8	10
Labour hours	6	5
Profit (Rs)	140	100

According to the market demand, at least 60 units from each quality must be produced per day. On each day, the manufacturer has 2400 kg of raw material and 1500 labour hours.

It is required to find the number of units to be produced per day from each quality to maximize the total profit.

- Formulate this as a linear programming problem.
- Sketch the feasible region.
- Using the graphical method, find the optimum solution for this problem.
- During the production of 2 units of Grade II, if 1 unit of a by-product is obtained that gives a profit of Rs. 80 per unit, find the optimum solution under this new situation.

12. (a) Let  $A = \begin{pmatrix} 3 & a \\ -4 & 3 \end{pmatrix}$  and  $B = \begin{pmatrix} 4 & b \\ 0 & 2 \\ 1 & 1 \end{pmatrix}$ , where  $a, b \in \mathbb{R}$ .

Find the values of  $a$  for which  $A^{-1}$  exists.

Now, let  $a = -2$ . Write down  $A^{-1}$ .

Let  $C = BA^{-1}$ . Find  $C$  in terms of  $b$ .

Find the value of  $b$  such that  $C^T B = \begin{pmatrix} 39 & 15 \\ 25 & 12 \end{pmatrix}$ .

(b) Let  $a, b \in \mathbb{R}$ . Write down the system of linear equations

$$ax - y = 2$$

$$4x - 2y = b$$

in the form  $\mathbf{AX} = \mathbf{B}$ , where  $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ , and  $\mathbf{A}$  and  $\mathbf{B}$  are matrices to be determined.

Show that the above system of equations has

- (i) a unique solution, when  $a \neq 2$ ,
- (ii) infinitely many solutions, when  $a = 2$  and  $b = 4$ ,
- (iii) no solutions, when  $a = 2$  and  $b \neq 4$ .

13.(a) An unbiased die with faces marked 1, 1, 2, 3, 4, 4 is rolled and an unbiased spinner with six separated regions numbered 1, 2, 3, 2, 2, 1 is spun.

The number on the top face of the die and the number on the region pointed by the arrow of the spinner are recorded.

Let  $A$  be the event that both recorded numbers are the same and  $B$  be the event that the sum of the two recorded numbers is even.

Find  $P(A')$ ,  $P(A' \cap B)$  and  $P(A' \cup B)$ .

(b) Numbers consisting of 5 digits are made by selecting digits from the seven digits from 1 to 7 without repeating any digit.

Find

- (i) the total number of different numbers that can be made,
- (ii) the number of different numbers that can be made starting with 3, and having 6 and 7 adjacent to each other.

(c) A mathematics society consists of 9 senior members and 6 junior members. A research team of 5 people is to be formed to work on a new project.

Find the number of different ways in which the research team can be made

- (i) out of all members,
- (ii) such that the team has at least 3 senior members.

14. The probability that a school bus arrives on time on a Monday is 0.7. If the bus arrives on time on a particular day, the probability that it will be on time on the following day is 0.8. If the bus is late on a particular day, the probability that it will be late on the following day is 0.4.

Find the probability that the bus arrives on time on a Wednesday.

Also, find the probability that, from Monday to Wednesday of a week, the bus will be

- (a) late at least on one day,
- (b) late exactly on two days,
- (c) late exactly on two days, given that it was late at least on one day.

[See page nine

15. The actual weight of cashews in packets labelled as 50 g of a certain cashew distributor follows a normal distribution with a mean of 48 g and a standard deviation of 4 g. A packet of cashews is randomly selected. Let  $A$  be the event that the actual weight of cashews in the chosen packet is less than the labelled weight. Let  $B$  be the event that the difference between the actual weight of cashews in the chosen packet and the labelled weight is at most 2 g.

Find,

- (i)  $P(A)$ ,
  - (ii)  $P(B)$ ,
  - (iii)  $P(A \cap B')$ ,
  - (iv)  $P(A|B)$ ,
- and (v)  $P(A \cup B)$ .

16. The following table summarises the distances travelled by a taxi driver during a certain period of time:

Distance (km)	No. of days
10 – 20	9
20 – 30	13
30 – 40	$a$
40 – 50	16
50 – 60	15
60 – 70	5

- (i) If the cumulative frequency corresponding to the 4<sup>th</sup> class interval is 60, find the value of  $a$ .
- (ii) Using the transformation  $y_i = \frac{x_i - 35}{10}$ , or otherwise estimate
  - (a) total distance travelled, during this period of time,
  - (b) mean of the distance travelled per day,
  - (c) median of the distances travelled during this period of time.
- (iii) Based on the values of the mean and the median, what can be concluded about the shape of the distribution of the data?
- (iv) Sketch the cumulative frequency curve.
- (v) Using the curve sketched in part (iv), or otherwise, estimate
  - (a) the tenth percentile,
  - (b) the first quartile,
  - (c) the inter-quartile range
 for the given data.

17. The duration of activities of a project and the flow of activities are given in the following table:

Activity	Preceding Activity/ Activities	Duration (in months)
A	–	3
B	–	4
C	A, B	5
D	A, C	2
E	C	3
F	B, C	4
G	D, E	5
H	E, F	6
I	G, H	1

- (i) Construct the project network.
- (ii) Prepare an activity schedule that indicates earliest start time, earliest finish time, latest start time, latest finish time and the float for each activity.
- (iii) Find the total duration of the project.
- (iv) Write down the critical path of the project.
- (v) What are the activities that can be delayed without extending the total duration of the project?
- (vi) How is the project completion time affected by delaying the activity F by 2 months?

\*\*\*