

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
 இலங்கைப் பரீட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம் இலங்கைப் பரීட்சைத் திணைக்களம்
 Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka
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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2025
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2025
 General Certificate of Education (Adv. Level) Examination, 2025

උසස් ගණිතය I
 உயர் கணிதம் I
 Higher Mathematics I

11 E I

පැය තුනයි
 மூன்று மணித்தியாலம்
 Three hours

අමතර කියවීමේ කාලය - මිනිත්තු 10 යි
 மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
 Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number

Instructions:

- * This question paper consists of two parts.
Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17).
- * **Part A:**
 Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- * **Part B:**
 Answer **five** questions only. Write your answers on the sheets provided.
- * At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- * You are permitted to remove **only Part B** of the question paper from the Examination Hall.

For Examiners' Use only

(11) Higher Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner	
Checked by:	1
	2
Supervised by:	

1. Factorize: $a^3 + 8b^3 + 27c^3 - 18abc$.

2. Let a relation R be defined on the set of all integers $\mathbb{Z}$ by aRb if $a^3 + b^3$ is divisible by 2. Show that R is an equivalence relation on $\mathbb{Z}$ and write down the equivalence class of 0.

3. Let $f(x) = \frac{1+x}{x-1}$ for $x \neq 1$. Show that f is one-to-one and find the range of f .
Also find $f^{-1}(1 + f^{-1}(x))$.

4. Let $a, b, c \in \mathbb{R}$. Show that
$$\begin{vmatrix} 2 & 2 & 2 \\ a+1 & b+1 & c+1 \\ a^4 & b^4 & c^4 \end{vmatrix} = (a-b)(b-c)(c-a)[(a+b)^2 + (b+c)^2 + (c+a)^2].$$

7. Find the constants a and b such that the function $f(x) = \begin{cases} \ln x + a & \text{if } 0 < x < b \\ 2 \tan^{-1}(x) & \text{if } x \geq b \end{cases}$ is differentiable everywhere on $(0, \infty)$.

8. Solve the differential equation $(1 + x^3) \frac{dy}{dx} + x^2 y = 4x^2$.

9. Let f be a real-valued function such that f'' is continuous on $[0, 1]$. Show that

$$\int_0^1 (xf''(x) + (1+x)f'(x) + f(x))dx = f'(1) + f(1).$$

10. Sketch the curves whose polar equations are $r = 2(\cos \theta + \sin \theta)$ and $r(\cos \theta + \sin \theta) = 1$, in the same diagram.

Find the polar coordinates of the points of intersection of these curves.

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 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2025
 General Certificate of Education (Adv. Level) Examination, 2025

උසස් ගණිතය I
 உயர் கணிதம் I
 Higher Mathematics I

11 E I

Part B

* Answer five questions only.

11. (a) Let A , B , C and D be subsets of a universal set S . Stating clearly the laws of Algebra of sets that you use, show that

$$(i) (A' \cap C) \cup (B' \cap C) \cup (C \cap D) = ((A \cap B)' \cup D) \cap C,$$

$$(ii) (A \cap B \cap C \cap D') \cup (A' \cap C) \cup (B' \cap C) \cup (C \cap D) = C.$$

- (b) A survey of 1000 women in a town was conducted about their hobbies. The result shows that 48% enjoyed gardening, while 32% enjoyed only gardening. 42% enjoyed reading novels, while 28% enjoyed only reading novels. 32% enjoyed painting, while 17% enjoyed painting only. It was observed that 200 women have more than one of these hobbies.

- (i) Find the number of women who enjoy all three hobbies.
 (ii) Find the number of women who do not enjoy any of these hobbies.

12. (a) Let a , b , c be positive real numbers.

Show that

$$(i) \frac{a+b}{2} \geq \sqrt{ab},$$

$$(ii) \frac{bc}{a} + \frac{ca}{b} + \frac{4ab}{c} \geq 2a + 2b + c, \text{ and}$$

$$(iii) \text{ if } a + b + c = x, \text{ then } 8abc \leq (x-a)(x-b)(x-c) \leq \frac{8x^3}{27}.$$

$$(b) \text{ Let } \mathbf{A} = \begin{pmatrix} 2 & 10 \\ 3 & 9 \end{pmatrix}.$$

Find the values of λ such that $|\mathbf{A} - \lambda \mathbf{I}| = 0$, where $\mathbf{I}$ is the 2×2 identity matrix.

Hence or otherwise, find the equations of the lines through the origin which gets mapped

onto itself, under the transformation defined by, $\begin{pmatrix} x' \\ y' \end{pmatrix} = \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix}.$

13. State and prove **De Moivre's Theorem** for a positive integral index.

Let $\omega = \cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)$. Show that $\omega^9 = 1$.

Write down the nine distinct roots of $z^9 = 1$.

Show that $\sum_{k=1}^8 \omega^k = -1$.

Hence, show that $\sum_{k=1}^8 \cos\left(\frac{2k\pi}{9}\right) = -1$.

Deduce that $\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right) = 0$.

14. (a) Let C_1 and C_2 be the two curves given by $y = \frac{x}{\sqrt{x^2+1}}$ and $y = \sqrt{x^2+1}$, respectively. Sketch the graphs of C_1 and C_2 in a same diagram.

Let S be the region enclosed by the two curves C_1 and C_2 and the lines $x = 0$ and $x = 1$.

Find the volume of the solid generated by rotating the region S about the y -axis through 2π radians.

- (b) Solve the differential equation:

$$\frac{dy}{dx} = \frac{x+y+2}{x-y+6}$$

15. (a) Let $a > 0$ and let $I_n = \int_0^x \frac{1}{(t^2+a^2)^n} dt$ for $x > 0$ and $n = 0, 1, 2, \dots$.

Show that $2na^2 I_{n+1} = (2n-1)I_n + \frac{x}{(x^2+a^2)^n}$.

Let $J_{n,m} = \int_0^x \frac{t^m}{(t^2+a^2)^n} dt$ for $x > 0$ and $m, n = 0, 1, 2, \dots$.

Show that $J_{n,m+2} + a^2 J_{n,m} = J_{n-1,m}$.

Deduce that $\int_0^a \frac{t^2}{(t^2+a^2)^2} dt = \frac{\pi-2}{8a}$.

- (b) Find the Maclaurin series of $\sin x$.

Deduce the Maclaurin series of $\sin(\sin x)$ in ascending power of x up to and including the term in x^3 .

Using this, find approximate values for

(i) $x \sin(\sin x) + 2$ at $x = 0.3$, and

(ii) $\int_0^1 (x + 2 \sin(\sin x^2)) dx$.

16. Let (h, k) be the mid-point of a chord of the circle $x^2 + y^2 = a^2$, where $a > 0$. Prove that the equation of the chord is given by $hx + ky = h^2 + k^2$.

Show that the equation of the locus of the mid-points of the chords of the circle $x^2 + y^2 = 16$ which are tangent to the hyperbola $9x^2 - 16y^2 = 144$ is given by $(x^2 + y^2)^2 = 16x^2 - 9y^2$.

17. (a) Let $f(x) = \frac{2 - \cos 2x}{3 - \sin 2x - \cos 2x}$ for $x \in \mathbb{R}$.

(i) Show that $\frac{3}{7} \leq f(x) \leq 1$ for $x \in \mathbb{R}$.

(ii) Solve the equations $f(x) = 1$ and $f(x) = \frac{3}{7}$, and find the coordinates of the turning points of $y = f(x)$ for $0 \leq x \leq \frac{3\pi}{2}$.

- (b) The following table gives the values of the function $f(x) = \ln(1 + x^2 + x^3)$ correct to two decimal places for the given values of x :

x	0.00	0.25	0.50	0.75	1.00
$f(x)$	0.00	0.08	0.32	0.69	1.10

Using **Simpson's Rule**, find an approximate value for $\int_0^1 f(x) dx$.

Hence, find an approximate value for $\int_0^1 \ln[(1 + x^2 + x^3)e^{x^3}] dx$.

Department Of Examinations , Sri Lanka

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2025
 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2025
 General Certificate of Education (Adv. Level) Examination, 2025

උසස් ගණිතය II
 உயர் கணிதம் II
 Higher Mathematics II

11 E II

පැය තුනයි
 மூன்று மணித்தியாலம்
 Three hours

අමතර කියවීමේ කාලය - මිනිත්තු 10 යි
 மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
 Additional Reading Time - 10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number

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Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17).
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- * **Part B:**
 Answer **five** questions only. Write your answers on the sheets provided.
- * At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- * You are permitted to remove **only Part B** of the question paper from the Examination Hall.
- * Statistical Tables will be provided.
- * g denotes the acceleration due to gravity.

For Examiners' Use only

(11) Higher Mathematics II		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
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	14	
	15	
	16	
	17	
	Total	

Total

In Numbers

In Words

Code Numbers

Marking Examiner

Checked by:

1

2

Supervised by:

Part A

1. Let l_1 be the line given by $\mathbf{r} = \mathbf{i} + \mathbf{j} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k})$, where $\lambda \in \mathbb{R}$. Also let l_2 be the line passing through the point $P(5, 0, \alpha)$ and parallel to the line l_1 . A couple with its vector moment $-3\mathbf{j} - 3\mathbf{k}$ comprises a force $\mathbf{F}$ acting along the line l_1 and a force $-\mathbf{F}$ acting along the line l_2 . Find the force $\mathbf{F}$ and the value of α .

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2. Parallelogram $OACB$ has its sides determined by $\vec{OA} = a\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\vec{OB} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$, where $a > 0$. It is given that the area of the parallelogram is $\sqrt{35}$. Find the value of a .

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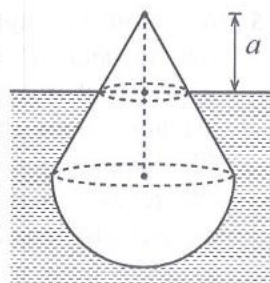
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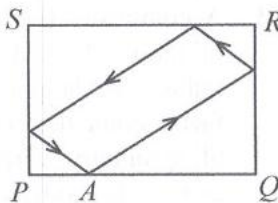
3. A composite body is formed by rigidly fixing a uniform solid hemisphere of radius $2a$ and density ρ to a uniform solid right circular cone of base radius $2a$, height $2a$ and density 2ρ in such a way that the two plane faces coincide. The composite body floats in equilibrium with its axis of symmetry vertical, in a liquid of density σ such that the part of the composite body, consisting of the hemisphere and the frustum of height a of the cone, is completely immersed in the liquid as shown in the figure.



Show that $\frac{\rho}{\sigma} = \frac{23}{32}$.

4. The position vector of a particle P at time t is given by $\mathbf{r}(t) = \sin(2t)\mathbf{i} + (\cos(2t) + 1)\mathbf{j} + 2\sin(t)\mathbf{k}$. Find the velocity vector, and the angle that it makes with the position vector. Also, show that the angle between the position vector and the acceleration vector is always obtuse.

5. A smooth rectangular horizontal plane is surrounded by smooth walls PQ , QR , RS and SP , where $PQ = a$ and $QR = b$. (See the figure.) A particle is projected from a point A on PQ with initial velocity $u\mathbf{i} + v\mathbf{j}$ where $\mathbf{i}$ is the unit vector in direction PQ and $\mathbf{j}$ is the unit vector in the direction PS . Suppose the particle, after rebounding from the walls QR , RS and SP , returns to the initial point A . The coefficient of restitution between the particle and the walls is e . Show that the time taken for the particle to move from A and return to A after rebounding at the walls QR , RS and SP is $\frac{b}{ev}(1+e)$.



6. A pendulum is constructed by rigidly attaching a thin uniform rod, of length $2l$ and mass m to a uniform disc, of radius l and mass $4m$, so that the rod lies in the plane of the disc and one end of the rod is at the centre of the disc. The pendulum is free to rotate in a vertical plane about a smooth horizontal axis through the other end of the rod perpendicular to the disc.

If the pendulum is released from rest with the rod horizontal, show that, when the rod makes an angle θ with the downward vertical, the angular velocity $\dot{\theta}$ is given by $\dot{\theta}^2 = \frac{27g \cos \theta}{29l}$.



7. The probability that a biased coin lands with a 'head' facing up and a 'tail' facing up are 0.4 and 0.6 respectively. In three independent tosses of this coin, find the probability that

- (i) a 'head' will turn up in at least one toss,
 (ii) two consecutive tosses will have 'head' turning up.

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8. The number of printing errors in a page of a book, X follows a Poisson distribution with probability mass function

$$P(X=x) = \frac{e^{-2} 2^x}{x!}, \quad x = 0, 1, 2, \dots$$

Find the expected number of printing errors in a page.

Errors occur in different pages independently. Given that there is exactly two errors in total in the first two pages, find the probability that there are no errors in the first page.

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9. The probability density function $f(x)$ of a continuous random variable X is given by

$$f(x) = \begin{cases} kx(1-x)^2 & , \quad 0 \leq x \leq 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find the value of k , and the cumulative distribution function $F(x)$, of X .

Hence or otherwise find $P(X > 0.5)$.

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10. Let X be a discrete random variable that can only take the values 0, 1 and 2.

Given that $P(X > 0) = 0.5$ and $P(X = 1) = p$, show that $P(X = 0) = 0.5$ and $E(X) = 1 - p$, where $0 < p < 0.5$.

Also, find the mean of the random variable Y defined by $Y = 1 - X$.

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 Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka Department of Examinations, Sri Lanka
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 கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2025
 General Certificate of Education (Adv. Level) Examination, 2025

උසස් ගණිතය II
 உயர் கணிதம் II
 Higher Mathematics II

11 E II

Part B

* Answer five questions only.

11. The points $A(1, 0, 0)$, $B(0, 1, 0)$, $C(0, 0, 1)$ and $D(1, \alpha, \beta)$ form a skew quadrilateral $ABCD$. The forces $3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$, $-\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $-\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{F}$ act through the points A , B , C and D , respectively. If the system of forces is equivalent to a couple with its vector moment $11\mathbf{i} - 5\mathbf{j} - \mathbf{k}$, find the force $\mathbf{F}$ and the values of α and β .
- Now two forces $10\mathbf{i} + 16\mathbf{j} + 5\mathbf{k}$ and $2\mathbf{i} - \mathbf{j}$, acting through the points $(1, 1, 1)$ and $(5, 0, 0)$, respectively are introduced to the above system.
- Find the force acting through the origin needed to be added to the system so that the system is in equilibrium.
12. A lamina in the shape of a square of side a is immersed in a homogeneous liquid with its surface vertical, the top edge horizontal and the centre lying below a distance h from the free surface of the liquid. Show that the centre of pressure of the lamina is at a distance $\frac{a^2}{12h}$ below the centre of the lamina.
- A cubical tank of side $2a$ is completely filled with a homogeneous liquid of density ρ . The tank has a square lid of side $2a$ on one vertical face which is smoothly hinged along its upper horizontal edge. The lid is kept closed by a smooth latch along the lower edge. The tank is immersed vertically in another homogeneous liquid of density $\frac{\rho}{2}$ such that the centre of the lid is at a depth $d(>a)$ below the free surface of the outer liquid. Now, the latch is released. Show that the lid starts to open if $d < \frac{7a}{3}$.
13. A particle P is dropped from rest from a height H . Simultaneously, another particle Q is projected vertically upwards from the ground with an initial velocity u . Both particles move along the same vertical line under the influence of gravity and the air resistance of kv per unit mass, where k is a constant and v is the speed.
- Show that
- (i) if Q does not meet P , the time taken by Q to reach its maximum height is $T = \frac{1}{k} \ln \left(1 + \frac{ku}{g} \right)$,
- and the total distance travelled by both particles at any time $t \leq T$ is $\frac{u}{k} \ln(1 - e^{-kt})$.
- (ii) if the particles P and Q meet while Q is still ascending, then $H < \frac{u^2}{ku + g}$.

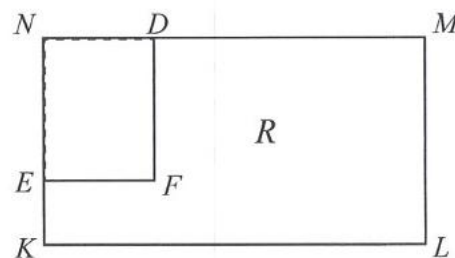
14. A smooth sphere A , of mass $2m$ travels along a straight line on a smooth horizontal table with speed u . It makes a perfectly elastic collision with another smooth sphere B of mass m and radius equal to that of A which is at rest on the table. Just before the collision, the velocity of A makes an angle ϕ ($0 < \phi < \frac{\pi}{2}$) with the line joining the centres of the spheres. Find the velocities of A and B just after the collision.

Let θ be the angle through which the direction of motion of A turns due to the impact.

Show that $\tan \theta = \frac{2}{3 \tan \phi + \cot \phi}$.

Determine the value of ϕ that makes θ a maximum.

15. A uniform rectangular lamina $KLMN$ is such that $KL = 12a$ and $KN = 8a$. The point D on MN is such that $ND = 4a$ and the point E on NK is such that $NE = 6a$. The mass per unit area of the lamina is ρ . The rectangle $NDFE$ is removed and let the remaining lamina be R . (See the diagram.) Show that the distance to the centre of mass of R from KL is $\frac{11}{3}a$ and from KN is $\frac{22}{3}a$.



Show that the moment of inertia of R about an axis through K perpendicular to the lamina is $5856a^4\rho$.

The lamina R can rotate in a vertical plane about a fixed smooth horizontal axis through K . The lamina is released from rest in a position where E is vertically above K . Find, in terms of a and g , the angular velocity of the lamina at an instant when KL is vertical.

16. Among customers who order rice at a restaurant, 60% order plain rice and 40% order yellow rice. Among those who order plain rice, 60%, 30% and 10% order lime juice, orange juice and mango juice respectively. Among those who order yellow rice, 70%, 25% and 5% order lime juice, orange juice and mango juice, respectively. Customers make choices independently. Each customer order only one type of rice and one type of juice.

Find the probability that a customer will order yellow rice given that he has ordered lime juice.

Now, consider two customers. Find the probability that

- both will order the same type of juice.
- they will order different types of rice.
- they will order the same type of juice and different types of rice.
- at least one of the customers will order yellow rice and orange juice.

17. (a) A random variable X follows an exponential distribution with the probability density function $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$; where $\lambda (> 0)$ is a parameter. Show that the cumulative distribution function $F(x)$ of X is given by $F(x) = 1 - e^{-\lambda x}$.

The lifetime of a certain type of air conditioners follows an exponential distribution. It is known that 19% of air conditioners of this type fails before 2 years of operation.

- (i) Find the percentage of air conditioners that will fail before one year of operation.
 - (ii) Find the probability that an air conditioner fails between two and three years of operation.
 - (iii) Given that an air conditioner did not fail during the first year of operation, find the probability that it will not fail before two years of operation.
- (b) A random variable X follows a normal distribution with a mean of 10 units and a standard deviation of 2 units. Given that a randomly chosen value from this distribution is greater than 10 units, find the probability that this value is in between 12.6 and 13.0 units.

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