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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2026
கல்விப் பொதுத் தராதரப் பத்திரி (உயர் தர)ப் பரீட்சை, 2026
General Certificate of Education (Adv. Level) Examination, 2026

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

Part B

※ Answer **five** questions only.

11. (*a*) Let $k, \lambda \in \mathbb{R}$ and $k \neq 0$. Show that the equation $kx^2 - 2\lambda x - \frac{1}{k} = 0$ has two non-zero real roots that are distinct.

Let α and β be these roots. Show that the quadratic equation with roots $\frac{1}{\alpha} + k^2\beta^2$ and $\frac{1}{\beta} + k^2\alpha^2$ is given by $f(x) = 0$, where $f(x) = x^2 - 2(2\lambda^2 - k\lambda + 1)x + (2k\lambda - k^2 + 1)$.

It is given that the axis of symmetry of the graph of $y = f(x)$ is $x = -1$. Show that $2\lambda^2 - k\lambda + 2 = 0$.

Find the positive value of k such that the equation $2\lambda^2 - k\lambda + 2 = 0$ is satisfied by only one value of λ and also find this value of λ.

(*b*) Let $f(x) = x^4 + ax^3 + px^2 + q$ and $g(x) = x^4 + bx^3 - qx^2 + 2x - p$, where $a, b, p, q \in \mathbb{R}$. It is given that $(x+1)$ is a factor of $f(x)$ and that the remainder when $g(x)$ is divided by $(x-1)$ is 4.

Show that $a = b$.

It is also given that the equation $f(x) = f(-x)$ has a non-zero solution. Show that $a = 0$ and $p + q = -1$.

Hence, show that $(x+1)$ is a factor of $g(x)$.

12. (*a*) Numbers are to be formed using some or all of the given five digits 3, 5, 6, 7 and 8 without repetition.

Find the number of different ways in which,

(i) a number greater than 10 000 can be formed,

(ii) a number greater than 6000 can be formed.

(*b*) (i) Find the real constants A, B and C such that $\dfrac{r+5}{(r+1)(r+2)(r+3)}=\dfrac{A}{r+1}+\dfrac{B}{r+2}+\dfrac{C}{r+3}$ for $r\in\mathbb{Z}^+$.

(ii) Let $U_r=\dfrac{r+5}{(r+1)(r+2)(r+3)}$ for $r\in\mathbb{Z}^+$ and $f(r)=\dfrac{1}{r+2}$ for $r\in\mathbb{Z}^+\cup\{0\}$.

Using part (i) above, show that

$U_r=2f(r-1)-3f(r)+f(r+1)$ for $r\in\mathbb{Z}^+$.

Hence, show that $\sum\limits_{r=1}^{n}U_r=\dfrac{2}{3}-\dfrac{2}{n+2}+\dfrac{1}{n+3}$.

Show that the infinite series $\sum\limits_{r=1}^{\infty}U_r$ is convergent and find its sum.

Find the value of $\lim\limits_{n\to\infty}\left(\sum\limits_{r=2}^{n+2}U_r+\sum\limits_{r=n+1}^{2n}U_r\right)$.

13.(*a*) Let $\mathbf{A}=\begin{pmatrix}2 & a & 1\\ 3 & -1 & 1\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}-2 & 0\\ b & 2\\ 2 & 3\end{pmatrix}$ and $\mathbf{C}=\begin{pmatrix}-3 & -3\\ 5 & c\end{pmatrix}$, where $a,b,c\in\mathbf{R}$.

Find the values of a, b and c, such that $\mathbf{AB}=\mathbf{C}^{\mathrm{T}}$.

With these values of a, b and c, find the matrix $\mathbf{D}$ is such that $(\mathbf{AB})^{\mathrm{T}}\mathbf{D}=2\mathbf{I}+\mathbf{C}$, where $\mathbf{I}$ is the identity matrix of order 2.

(*b*) Let $z_1,z_2\in\mathbb{C}$. Show that

(i) $\overline{z_1+z_2}=\overline{z}_1+\overline{z}_2$, (ii) $\overline{z_1z_2}=\overline{z}_1\,\overline{z}_2$,

(iii) $\overline{\left(\dfrac{z_1}{z_2}\right)}=\dfrac{\overline{z}_1}{\overline{z}_2}$ for $z_2\neq 0$, and (iv) $z_1\overline{z}_1=|z_1|^2$.

Show that if $|z_1|=|z_2|=1$ and $z_1\neq -z_2$, then $\left(\dfrac{1+z_1z_2}{z_1+z_2}\right)$ is real.

(*c*) Show that $\dfrac{\cos\alpha+i\sin\alpha}{\cos\beta+i\sin\beta}=\cos(\alpha-\beta)+i\sin(\alpha-\beta)$, where $\alpha,\beta\in\mathbf{R}$.

Express $1+\sqrt{3}i$ and $\sqrt{2}(1+i)$ in the form $r(\cos\theta+i\sin\theta)$, where $r>0$ and $-\pi<\theta\leq\pi$.

Let $\omega=\dfrac{1+\sqrt{3}i}{\sqrt{2}(1+i)}$. Show that $|\omega|=1$ and Arg $\omega=\dfrac{\pi}{12}$.

Deduce that $\omega^{18}=-i$.

14. (*a*) Let $f(x)=a\left(\dfrac{x-1}{x+1}\right)^2+4$ for $x\in\mathbb{R}-\{-1\}$, where $a\in\mathbb{R}$. Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x)=\dfrac{4a(x-1)}{(x+1)^3}$ for $x\in\mathbb{R}-\{-1\}$.

It is given that the gradient of the tangent line to the graph of $y=f(x)$ when $x=0$ is 8. Show that $a=-2$.

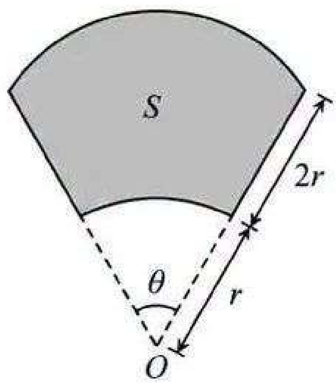
Find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

Also, find the coordinates of the turning point of the graph of $y=f(x)$.

Sketch the graph of $y=f(x)$ indicating the turning point and the asymptotes.

Hence, state the range of f and the smallest value of k such that f is one-to-one on the interval $[k,\infty)$.

(*b*) As shown in the figure, the boundary of the shaded region S consists of two concentric circular arcs of radii r m and $2r$ m subtending an angle θ radians at the centre O and two straight line segments when extended pass through the centre O. The area of the region S is given to be 81 m^2. Show that the perimeter p of S is given by $p=2\left(r+\dfrac{81}{r}\right)$ for $r>0$ and find the values of r and θ such that p is minimum.



15. (*a*) Express $\dfrac{6t^2+7t+3}{(2t-1)(4t^2+4t+5)}$ in partial fractions and **hence,** find $\displaystyle\int\frac{6t^2+7t+3}{(2t-1)(4t^2+4t+5)}\,\mathrm{d}t$.

(*b*) (i) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\tan\dfrac{x}{2}\right)=\dfrac{1}{1+\cos x}$.

(ii) Show that $\displaystyle\int_0^{\frac{\pi}{2}}\frac{x^2\sin x}{(1+\cos x)^2}\,\mathrm{d}x=\frac{\pi^2}{4}-2\int_0^{\frac{\pi}{2}}\frac{x}{1+\cos x}\,\mathrm{d}x$.

Hence, using (i), show that $\displaystyle\int_0^{\frac{\pi}{2}}\frac{x^2\sin x}{(1+\cos x)^2}\,\mathrm{d}x=\frac{\pi}{4}(\pi-4)+2\ln 2$.

(*c*) Show that $\cot\left(\dfrac{3\pi}{4}-x\right)=\dfrac{1-\cot x}{1+\cot x}$.

Using the formula $\displaystyle\int_a^b f(x)\,\mathrm{d}x=\int_a^b f(a+b-x)\,\mathrm{d}x$, where a and b are constants,

show that $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\ln(1+\cot x)\,\mathrm{d}x=\frac{\pi}{8}\ln 2$.

16. (*a*) Let $A=(3,0)$, $B=(0,2)$ and l be a line perpendicular to the line AB. Show that the equation of the line l has the form $3x-2y=\lambda$, where $\lambda \in \mathbb{R}$. This line l intersects the x-axis at the point C and the y-axis at the point D. Show that the point of intersection of the lines AD and BC lies on the circle $x^2+y^2-3x-2y=0$.

(*b*) Let $S_1 \equiv x^2+y^2-6x-4y+4=0$ and $S_2 \equiv x^2+y^2+4y-5=0$. Show that the circles $S_1=0$ and $S_2=0$, have equal radii and they intersect each other.

Also, find the equations of the common tangents to these circles.

Further, let $U \equiv 2x-4y+3$. Find the equation of the circle that passes through the points of intersection of the circles $S_1+U=0$ and $S_2=0$, and intersects the circle $S_2=0$ orthogonally.

17. (*a*) Express $\cos 2x+\sin 2x$ in the form $R\sin(2x+\alpha)$, where $R>0$ and $0<\alpha<\dfrac{\pi}{2}$.

Hence, find the general solution of the equation $1+\cos 2x+\sin 2x=0$.

Let S be the set consisting of these solutions.

Show that $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}=\tan x$ for $x \notin S$.

Hence, solve $\dfrac{1-\cos x+\sin x}{1+\cos x+\sin x}=\dfrac{1}{2}\sin\dfrac{x}{2}$.

(*b*) In the usual notation, state the **Cosine Rule** for a triangle ABC.

In a triangle ABC, the point D lies on BC such that $BD:DC=2:1$.

Applying the Cosine Rule for suitable triangles,

show in the usual notation that $AD^2=\dfrac{1}{9}\left(6b^2+3c^2-2a^2\right)$.

Now, if $AB=3$ cm, $AC=2$ cm and $AD=\dfrac{\sqrt{37}}{3}$ cm, show that $B\hat{A}C=\dfrac{\pi}{3}$.

(*c*) Solve the equation $\sin\left(\tan^{-1}\left(\dfrac{1}{\sqrt{x^2+1}}\right)\right)+\tan\left(\sin^{-1}\left(\dfrac{1}{\sqrt{x^2+1}}\right)\right)=\dfrac{3}{2x}$.