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கல்வி, உயர் கல்வி மற்றும் தொழிற் கல்வி அமைச்சு
Ministry of Education, Higher Education and Vocational Education

G.C.E (A/L) Supportive Assesment - 2026

සංයුක්ත ගණිතය	II
இணைந்த கணிதம்	II
Combined Mathematics	II

10 E II

පැය තුනයි	
மூன்று மணித்தியாலம்	
Three hours	

අමතර කියවීම් කාලය	-	මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம்	-	10 நிமிடங்கள்
Additional Reading Time	-	10 minutes

Use additional reading time to go through the question paper, select the questions you will answer and decide which of them you will prioritise.

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
* **Part A:**
  Answer ***all*** *questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  Answer ***five*** *questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove* ***only*** **Part B** *of the question paper from the Examination Hall.*

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* In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
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	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

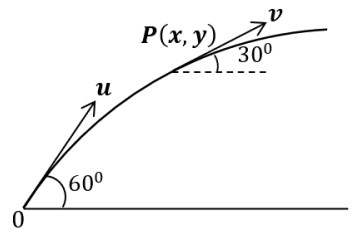
Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. A particle A of mass m and a particle B of mass $2m$, moving on a smooth horizontal table along the same straight line towards the same direction with velocities u and $v(< u)$ respectively, collide directly. The coefficient of restitution between A and B is e. Show that the momentum transferred from one particle to the other particle is given by $\frac{2m}{3}(1+e)(u-v)$.

2. A particle is projected from a point O on a horizontal ground, with an initial speed u and at an angle 60^0 to the horizontal. The particle passes through a point $P(x, y)$, which is located on OXY plane. If the direction of the motion of the particle makes an angle of 30^0 to the horizontal at P, then show that $\frac{x}{y} = \frac{\sqrt{3}}{2}$.

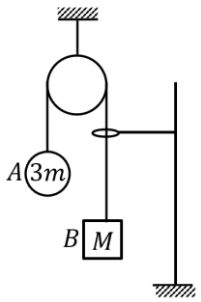


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[see page three

Index Number | | | | | | | |

3. Two particles, A and B, of masses $3m$ and M respectively, are connected by a light inextensible string which passes over a fixed smooth pulley and through a fixed smooth ring, as shown in the figure. The system is released from rest with the string taut. The parts of the string which are not in contact with the pulley are vertical. It is given that the particle B moves up with acceleration $\frac{g}{5}$. Show that $M = 2m$.

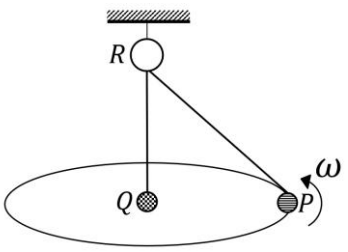


In the subsequent motion the particle B strikes on the ring with speed $\sqrt{\frac{2gl}{5}}$ and comes to the rest. Find the impulsive force exerted on the ring.

4. A car of mass M kg travels along a straight horizontal road against a resistive force proportional to $v^{\frac{3}{2}}$ N, where v is the speed of the car. The resistive force is given by $\mathrm{R} = \mathrm{k}\, v^{\frac{3}{2}}$, where k is a constant. The maximum speed of the car on the horizontal road is $25\ \mathrm{m\ s^{-1}}$, and the power of the engine is 25 kW. Show that $\mathrm{k} = 8$. Now, the engine is operating at lower power while the car descends a slope of inclination $\sin^{-1}\frac{1}{20}$ to the horizontal. Find the power of the engine of the car, at an instant when the speed of the car is $16\ \mathrm{m\ s^{-1}}$ and the acceleration of the car is $\frac{1}{2}\mathrm{m\ s^{-2}}$. (Take $g = 10\ \mathrm{m\ s^{-2}}$)

[see page four

5. Two particles, P and Q of masses m and $2m$ respectively are attached to the two ends of a light inextensible string of length l which passes through a small smooth ring R, as shown in the figure. The particle P moves in a horizontal circle with a constant angular velocity ω while the string remains taut. Particle Q is at rest at the center of the circular path of P. Show that $\omega = \sqrt{\frac{3g}{l}}$. **Hence,** find the radius of the circular path of P in terms of l.

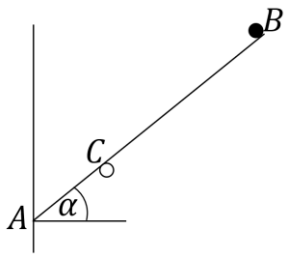


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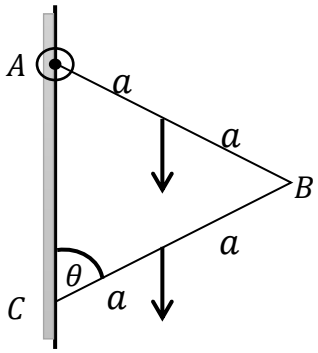
6. Let O, A and B are three collinear points and $\lambda \in \mathbb{R}$. In the usual notation, the position vectors of two points A and B, with respect to a fixed origin O, are $\mathbf{i}+\lambda\mathbf{j}$ and $3\mathbf{i} + 6\mathbf{j}$, respectively. Find the value of λ. Let P be the point on y-axis such that the lines AP and BP are perpendicular to each other. Find the two possible position vectors for P.

[see page five

7. A uniform rod ACB of length $4a$ and weight w is kept in equilibrium with the end A against a smooth vertical wall and end B attached to a particle of weight $2w$, by a smooth peg placed at C, such that $AC = a$. The rod makes an angle α with the horizontal, as shown in the figure, where $\tan\alpha = \sqrt{\frac{7}{3}}$. Find the magnitudes of the reactions at A and C in terms of w.



8. Two uniform rods AB, BC each of length $2a$ and weight W are smoothly jointed at B and the end A hinged to a rough vertical wall. The system rests in equilibrium in a vertical plane perpendicular to the wall with the end C of the rod BC inclined at an angle θ to the vertical. The coefficient of friction between the rod BC and the wall is μ . Show that $\tan\theta \geq \frac{2}{\mu}$.



[see page six

9. Let A, B and C be three events of a sample space Ω. In the usual notation, it is given that $P(B) = \frac{3}{10}$ and $P(A \cup B) = \frac{3}{5}$. Find $P(A \cap B')$. If $(A' \cap B)$ and C are mutually exclusive and exhaustive events and $P(A) = \frac{1}{2}$, show that $P(C) = \frac{9}{10}$.

10. The mean of a set of five observations each of which is a positive integer is 4. The unique mode of the set is 3 and the range of the observations is 5. **Given that** the median is 3, find two possible sets of observations.

[see page seven

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இணைந்த கணிதம்	II
Combined Mathematics	II

10 E II

✱ Answer **five** questions only.

Part B

(In this question paper, g denotes the acceleration due to gravity.)

11. (a) A motor car X, moving along a straight road, passes a point A with velocity u and a uniform acceleration f. It accelerates until its velocity becomes λu, where $(\lambda > 2)$. At a point B, it begins to move with uniform deceleration f and reaches point C when its velocity is $2u$. Another motor car Y, moving along the same road in the same direction with uniform velocity of $2u$, passes the point A a time $\frac{u}{f}$ before motor car X passes same point A.

Sketch velocity-time graphs for the motions of both X and Y in the same diagram. **Hence,**

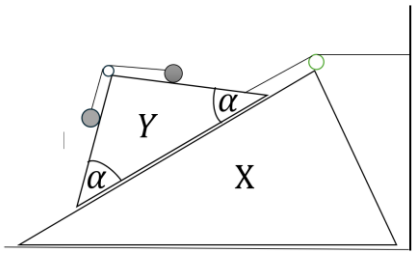
(i) show that $AB = \frac{1}{2f}u^2(\lambda^2 - 1)$

(ii) show that the total distance travelled by X, when it reaches the point C is $\frac{1}{2f}u^2(2\lambda^2 - 5)$

(iii) Show that if $\lambda \leq 2 + \frac{\sqrt{10}}{2}$, X cannot overtake Y.

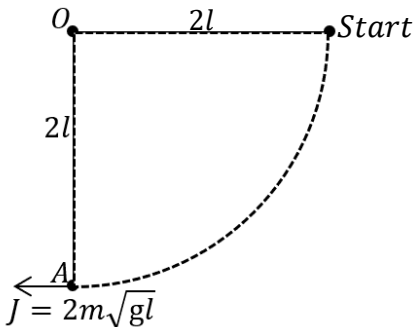
(b) A ship S is sailing with a constant velocity of v kmh^{-1} relative to the Earth in a direction making an angle α west of north. A warship W is sailing due west with a constant velocity of $u\ (< v \sin\alpha)$kmh^{-1} relative to the Earth. The warship detects ship S as an enemy vessel and attempts to intercept it. At a certain instant, the ship is directly south-west of the warship at a distance d km. By sketching the velocity triangle, determine the path of the ship relative to the warship. **Hence,** find the shortest distance l between the warship and the ship. If the maximum firing range of the guns fixed to the warship is $R\ (> l)$km, show that the ship remains within the firing range of the guns for a time interval of $\frac{2\sqrt{R^2 - l^2}}{\sqrt{v^2 + u^2 - 2uv\sin\alpha}}$ hours.

12. (a) The vertical cross-section through the centres of mass of two smooth uniform wedges, X and Y, particles P and Q is shown in the figure. The smooth wedge X, of Mass M is placed on a smooth horizontal table. On its inclined face, which makes an angle $\alpha\left(< \frac{\pi}{4}\right)$ with the horizontal, another smooth wedge Y, of mass $2m$, whose inclined faces make an angle α is placed. Two particles P and Q, each of mass m, are attached to the two ends of a light inextensible string which passes over a fixed smooth pulley at a vertex of wedge Y. One end of another light inextensible string, which passes over a fixed smooth pulley at a vertex of wedge X, is attached to wedge Y, and the other end is fixed to a point on a vertical wall in the same plane, such that the string remains horizontal. The system is released from rest with the strings taut. Write down equations sufficient to determine the tensions in the strings.



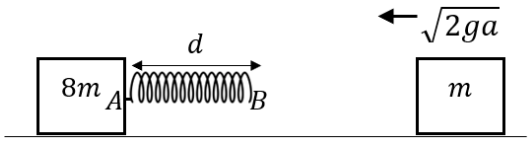
[see page eight

(b) As shown in the figure two ends of a light inextensible string of length $2l$ is attached to a small particle P of mass m and fixed-point O. The particle is released from rest at the same horizontal level of O with the string taut. In the subsequent motion particle P moves in a vertical circular path and reaches the lowest point A and then receives a horizontal impulse of magnitude $2m\sqrt{gl}$ in the direction of its motion. Find the velocity immediately after the impulse.



Show that the velocity and the tension in the string when OP make an angle 60^0 with the downward vertical are $\sqrt{14gl}$ and $\frac{15mg}{2}$ respectively. If at the same moment the string is suddenly cut without imparting any additional impulse and then the particle moves freely under the gravity and reaches point B which is at the same horizontal level of point A, show that the time taken by particle P to reach B is $\left(\frac{\sqrt{42}+5\sqrt{2}}{2}\right)\sqrt{\frac{l}{g}}$

13. A small cubical block of mass $8m$ is attached to one end A of a light elastic spring AB of natural length d and modulus of elasticity $6mg$. The spring and block are at rest on a horizontal table with AB equal to d and lying perpendicular to the face to which A is attached. Another block of equal physical dimensions, but of mass m, moving with a speed $\sqrt{2ga}$ in the direction parallel to BA impinges on the free end B of the spring.



It is assumed that the contact between the lighter block and the table is smooth.

Assuming that the heavier block is held fixed, AB remains straight and horizontal in the subsequent motion, when the one end of the spring B is at a distance x from A, Show that the equation of the motion of the lighter block is given by $\ddot{X} = -\omega^2 X$, where $X = x - d$

By assuming solutions for x in the form $x = d + h\cos\omega t + k\sin\omega t$, find the values of the constants h and k. **Hence,** Find the minimum length of the spring and **deduce** that $d > \frac{a}{3}$.

If $d = a$, show that the ratio between the time taken to compress the length of the spring to half of its length for the first time and time taken to reach the minimum length of the spring for the first time is 2: 3.

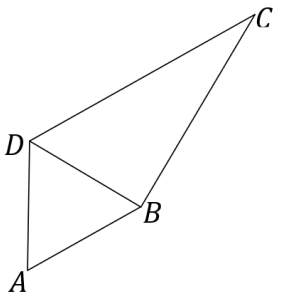
Now, the heavier block is free to move on the table and the coefficient of friction between the heavier block and the table is μ, find the least value of μ in terms of a and d in order to keep the heavier block not moving.

[see page nine

14. (a) Let $OACB$ be a parallelogram with O as the vector origin and **a** and **b** be two unit **vectors**. The position vectors of the vertices A and B with respect to an origin O, are $\alpha\mathbf{a}$ and $\beta\mathbf{b}$ respectively. The point P is on OA such that $OP : OA = \alpha : 1$ and the point Q is on AC such that $AQ : AC = \beta : 1$ Using the triangular law of vector addition, show that $\overrightarrow{PC} = \alpha(1-\alpha)\mathbf{a} + \beta\mathbf{b}$ and $\overrightarrow{BQ} = \alpha\mathbf{a} - \beta(1-\beta)\,\mathbf{b}$

The lines PC and BQ meet at R such that $B\hat{R}P = \frac{\pi}{2}$. Show that the angle θ, between the unit vector **a** and **b** is given by $\cos^{-1}\left(\frac{\beta^2(1-\beta) - \alpha^2(1-\alpha)}{\alpha\beta(\alpha+\beta-\alpha\beta)}\right)$.

(b) A quadrilateral $ABCD$ is shown in the figure, where AD is vertical. Triangle ABD is equilateral with side length $2a$, $C\hat{B}D = 90°$ and $CD = 4a$. Forces of magnitudes $6P, 4P, \alpha P, \beta P$ and $8\sqrt{3}P$ act along AB, AD, DC, DB and CB respectively in the directions indicated by the order of the letters where α and β are real constants.



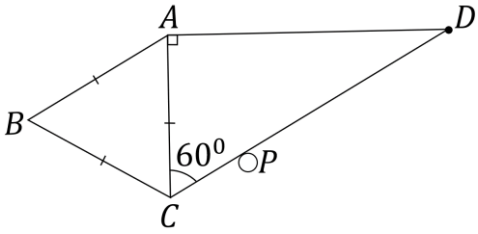
Given that the resultant of the system is parallel to DB in the sense from D to B. Show that $\alpha = 6$.

If the magnitude of the resultant force is $6P$, find the possible value for β.

Calculate the moment of the couple that must be added to the system so that it reduces to a single force along DB.

The resultant system is brought into equilibrium by applying a force F_1 at A, perpendicular to BD, and a force F_2 at C. Find the values of F_1 and F_2 .

15. (a) Uniform rods AB and BC each of length $2a$ and weight W and two other uniform rods AD and CD of weights W and $2W$ and of lengths $2\sqrt{3}a$ and $4a$ respectively are freely jointed at their ends. A light rod AC of length $2a$ is freely jointed to A and C. The end D is smoothly hinged to a fixed point and the rod CD is in contact with a small smooth peg P where $PD = 3a$. The system is in equilibrium in a vertical plane as shown in the figure with AD horizontal.

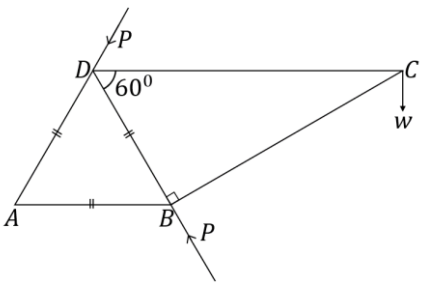


(i) Find the reaction at B on the rod AB

(ii) Show that the thrust in the light rod is $\frac{3W}{2}$

Also, find the reaction exerted on the rod CD by the peg P

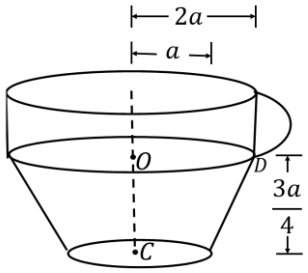
(b) The frame work shown in the figure consists of five light rods AB, BC, CD, DA and DB that are smoothly jointed at their ends. It is given that $AB = AD = BD = a$, $D\hat{B}C = 90^0$, $B\hat{D}C = 60^0$. A load w is suspended at the joint C and the frame work is smoothly hinged to a fixed point at A and kept in equilibrium in a vertical plane with AB and DC horizontal, by two forces each of magnitude P applied in $\overrightarrow{BD}$ and $\overrightarrow{DA}$ directions to it at the joints B and D respectively. Find the value of P. Draw a stress diagram using Bow's notation. **Hence,** find the stresses in the rods stating whether they are tensions or thrusts.



[see page ten

16. Show that the centre of mass of a thin uniform wire in the shape of a semicircular arc of radius a is at a distance $\frac{2a}{\pi}$ from its centre and that the centre of mass of a uniform hollow right circular cone of height h is at a distance $\frac{h}{3}$ from the centre of the base of the cone.

A vessel is formed by rigidly fixing a uniform thin shell in the shape of a frustum of hollow right circular cone, whose upper and lower circular rims have radii $2a$ and a respectively and height $\frac{3a}{4}$, to a thin cylindrical shell of radius $2a$ and height $\frac{a}{2}$ at the upper rim of the frustum and to a thin circular plate of radius a at the lower rim, as shown in the figure. A handle made of a thin semicircular wire of radius is fixed to the cylindrical part.



The mass per unit area of the frustum, the cylinder and the plate is ρ and the mass per unit length of the wire is $4a\rho$. Show that the centre of mass of the vessel is located at $\left(\frac{5a}{31}, \frac{2a}{31\pi}(4\pi+1)\right)$ on the plane of symmetry Oxy where the $\boldsymbol{x}$ axis lies along OC and the $\boldsymbol{y}$ axis lies along OD of which the origin is O.

It is suggested that a thin wire ring of radius $2a$ should be attached along the rim of the cylindrical shell such that OC remains horizontal when the vessel is suspended from handled by a smooth peg. Find the mass per unit length of the wire ring used.

17. (a) A certain athlete participates in a track event consisting of several rounds and attempts to qualify for the final round. In the preliminary round, the probabilities that he obtains the first, second, and third positions are $\frac{1}{5}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. If the athlete obtains 1st 2nd or 3rd in the preliminary round, the probability that he qualifies for the final round is $\frac{3}{5}$ in each case. Otherwise, the probability that he qualifies for the final round is $\frac{1}{10}$. Find the probability that the athlete qualifies for the final round. **Given that,** the athlete has qualified for the final round, find the probability that he obtained one of the first three positions in the preliminary round.

(b) Let x_i and f_i be the class mark and the frequency of the i^{th}class interval of a grouped data distribution. It is **given** that the mean $\bar{x}$ and standard deviation σ of the distribution are $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$ and $\sigma = \sqrt{\frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}}$ respectively. If the observations are transformed using $y_i = ax_i + b$ where $a, b \in \mathbb{R}$, Show that the mean and the standard deviation of transformed data distribution are given by $\bar{y} = a\bar{x} + b$ and $\sigma_y = |a|\sigma$ respectively.

The Adjoining tables show summarized data collected by a researcher from the first 30 customers of a food court who preferred Food Z, on a particular day, with the aim of assisting the owner in making decisions to improve sales. Find the mean and the standard deviation of the customer's ages. **Hence,** deduce the mean expenditure on food Z and the corresponding standard deviation.

Ages of Customers (years)	Number of customers: Food Z
61–75	2
46–60	2
31–45	8
16–30	12
1–15	6

Expenditure (rupees)	Number of customers:
6100–7500	2
4600–6000	2
3100–4500	8
1600–3000	12
1100–1500	6

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கல்வி, உயர் கல்வி மற்றும் தொழிற் கல்வி அமைச்சு

Ministry of Education, Higher Education and Vocational Education

Supportive Assessment – 2026

10 – Combined Mathematics

Marking Scheme – Paper II

Preface

It is with great pleasure that this **Marking Guide for the Supportive Assessment – 2026 in Combined Mathematics** is presented to students and teachers involved in the preparation for the forthcoming **G.C.E. Advanced Level Examination**.

This guide has been carefully prepared with the objective of supporting both the teaching and learning processes in Combined Mathematics. Every effort has been made to ensure that the solutions and approaches are consistent with the standards and principles followed in the G.C.E. (A/L) Examination.

While the guide is based on the structure and methodology of the official G.C.E. (A/L) marking scheme, it should be noted that the two serve different purposes. The official marking scheme primarily indicates the **essential mark-awarding steps** required for assessment. In contrast, this guide has been developed as a **student-friendly learning resource**, incorporating additional intermediate steps, explanations, and mathematical details wherever necessary to facilitate understanding.

Students should therefore understand that, in the actual G.C.E. (A/L) Examination, marks are awarded only for the essential steps and key mathematical arguments specified in the official marking scheme. The additional steps included in this guide are intended to clarify the solution process and strengthen conceptual understanding rather than represent separate mark-awarding points.

Teachers may also use this guide as a reference when marking answer scripts of the Supportive Assessment. The detailed presentation of solutions enables teachers to identify students' strengths and weaknesses more effectively while maintaining consistency with the expected examination standards.

Furthermore, special notes and observations have been included at appropriate places to highlight important concepts, examination techniques, common errors, and areas that require particular attention. It is hoped that these additions will help students develop a deeper understanding of the subject and improve the quality of their mathematical reasoning and presentation.

It is sincerely hoped that this guide will serve as a valuable resource for both students and teachers, contributing positively to the teaching, learning, and assessment of Combined Mathematics.

Best Wishes to Students

Success in Combined Mathematics is achieved through perseverance, disciplined practice, logical thinking, and a clear understanding of fundamental concepts. Every problem you attempt and every challenge you overcome contributes to your growth as a learner.

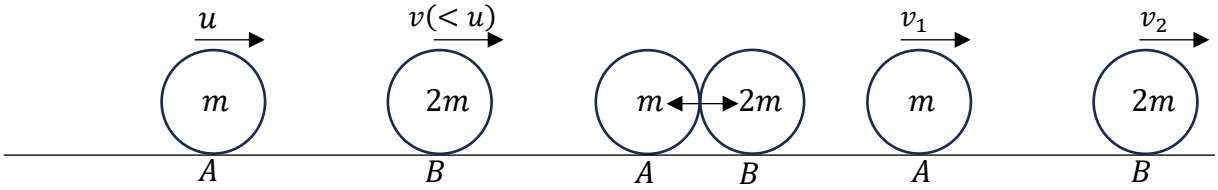
May this guide assist you in refining your problem-solving skills, improving your examination techniques, and gaining confidence in your mathematical abilities. As you prepare for the forthcoming G.C.E. Advanced Level Examination, we wish you determination, success, and excellence in all your academic endeavors.

May your hard work be rewarded with outstanding results and a bright future ahead.

Mathematics Branch
Ministry of Education, Higher Education and Vocational Education
2026.06.24

Part A

1. A particle A of mass m and a particle B of mass $2m$, moving on a smooth horizontal table along the same straight line towards the same direction with velocities u and $v(<u)$ respectively, collide directly. The coefficient of restitution between A and B is e. Show that the momentum transferred from one particle to the other particle is given by $\frac{2m}{3}(1+e)(u-v)$



(Before collision) (When collide) (After collision)

Let I be the momentum transferred from one particle to other

N.L. of Res: $v_2 - v_1 = -e(v-u)$ ——— (**1**) **5**

$I = \Delta(m\underline{v})$ for entire sys:

$0 = (mv_1 + 2mv_2) - (mu + 2mv)$

$v_1 + 2v_2 = u + 2v$ ——— (**2**) **5**

(**1**) + (**2**);

$3v_2 = u + 2v - e(v-u)$ **5**

$I = \Delta(m\underline{v})$ for B:

$I = \Delta m\underline{v}$ for B:

$I = 2mv_2 - 2mv$ **5**

$$I = 2m\left[\frac{u+2v-e(v-u)}{3} - v\right]$$

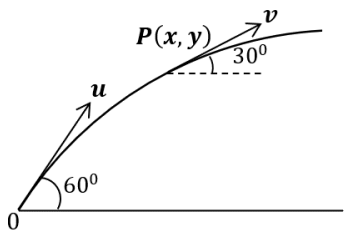
$$= \frac{2m}{3}[u + 2v - ev + eu - 3v]$$

$$= \frac{2m}{3}[u(1+e) - v(1+e)] \quad \textbf{5}$$

$$= \frac{2m}{3}(1+e)(u-v)$$

25

2. A particle is projected from a point O on a horizontal ground, with an initial speed u and at an angle 60^0 to the horizontal. The particle passes through a point $P(x, y)$, which is located on OXY plane. If the direction of the motion of the particle makes an angle of 30^0 to the horizontal at P, then show that $\frac{x}{y} = \frac{\sqrt{3}}{2}$.



$0 \rightarrow P \rightarrow \overrightarrow{S = ut + \frac{1}{2}at^2}$

$x = u\cos 60^0\ t$

$x = \frac{u}{2}t$ ——— **(1)**

5

$0 \rightarrow P \quad \overrightarrow{v = u + at}$

$v\cos 30^0 = u\cos 60^0$

$v \cdot \frac{\sqrt{3}}{2} = u \cdot \frac{1}{2}$

$v = \frac{u}{\sqrt{3}}$

$\uparrow s = ut + \frac{1}{2}at^2$

$y = u\sin 60^0\ t - \frac{1}{2}gt^2$

$y = \frac{\sqrt{3}}{2}ut - \frac{1}{2}gt^2$ ——— **(2)** **5**

$\uparrow v = u + at$

$v\sin 30^0 = u\sin 60^0 + (-g)t$

$v \cdot \frac{1}{2} = u \cdot \frac{\sqrt{3}}{2} - gt$

(1)/(2); $\frac{x}{y} = \frac{\frac{u}{2}t}{\frac{\sqrt{3}}{2}ut - \frac{1}{2}gt^2}$

$gt = u \cdot \frac{\sqrt{3}}{2} - v \cdot \frac{1}{2}$

$\frac{x}{y} = \frac{u}{\sqrt{3}u - gt}$ **5**

$gt = \frac{1}{2}(\sqrt{3}u - v)$ **5**

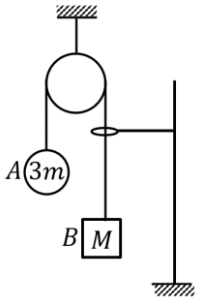
$\frac{x}{y} = \frac{u}{\sqrt{3}u - \frac{1}{2}\left(\sqrt{3}u - \frac{u}{\sqrt{3}}\right)}$ **5**

$gt = \frac{1}{2}\left(\sqrt{3}u - \frac{u}{\sqrt{3}}\right)$

$\frac{x}{y} = \frac{\sqrt{3}}{2}$

25

3. Two particles, A and B, of masses $3m$ and M respectively, are connected by a light inextensible string which passes over a fixed smooth pulley and through a fixed smooth ring, as shown in the figure. The system is released from rest with the string taut. The parts of the string which are not in contact with the pulley are vertical. It is given that the particle B moves up with acceleration $\frac{g}{5}$. Show that $M = 2m$.



In the subsequent motion the particle B strikes on the ring with speed $\sqrt{\frac{2gl}{5}}$ and comes to the rest. Find the impulsive force exerted on the ring.

$F = ma$

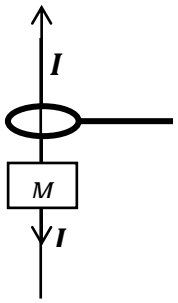
$(M) \uparrow;\quad T - Mg = M\left(\frac{g}{5}\right)$ ——— $(\mathbf{1})$ **5**

$(3m) \downarrow;\quad 3mg - T = 3m\left(\frac{g}{5}\right)$ ——— $(\mathbf{2})$ **5**

$(\mathbf{1}) + (\mathbf{2})$;For the particle B,

$(3m - M)g = \frac{g}{5}(M + 3m)$ **5**

$M = 2m$



$\uparrow I = \Delta(m\underline{v})$

$I = m\left(\sqrt{\frac{2gl}{5}} - 0\right)$ **5**

$I = m\sqrt{\frac{2gl}{5}}$ **5**

25

4. A car of mass M kg travels along a straight horizontal road against a resistive force proportional to $v^{\frac{3}{2}}$N, where v is the speed of the car. The resistive force is given by $R = k\,v^{\frac{3}{2}}$, where k is a constant. The maximum speed of the car on the horizontal road is $25\ \text{m s}^{-1}$, and the power of the engine is 25 kW. Show that k = 8. Now, the engine is operating at lower power while the car descends a slope of inclination $\sin^{-1}\frac{1}{20}$ to the horizontal. Find the power of the engine of the car, at an instant when the speed of the car is $16\ \text{m s}^{-1}$ and the acceleration of the car is $\frac{1}{2}\ \text{m s}^{-2}$.
(Take $g = 10\ \text{m s}^{-2}$)

$P = Fv$

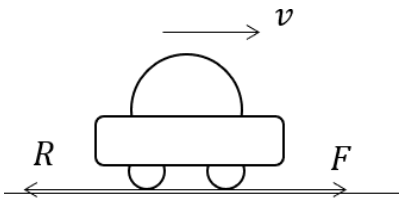
$25 \times 10^3 = F \cdot 25$ **5**

$\therefore F = 10^3\ N$

$\overrightarrow{F} = ma$

$\text{F} - \text{R} = M(0)$

$\text{F} = \text{R}$



Since $R = k\,v^{\frac{3}{2}}$

$10^3 = k\,(25)^{\frac{3}{2}}$ **5**

$\text{K} = 8$

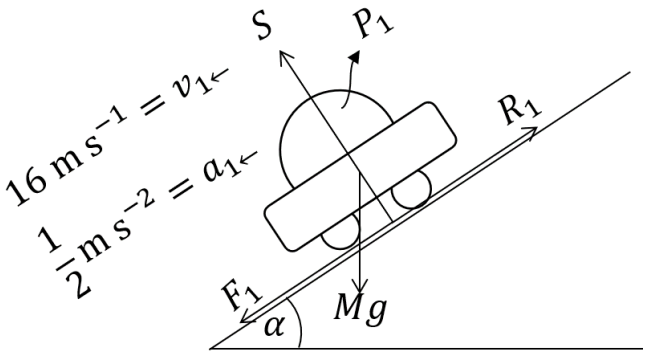
$F = ma$

$F_1 + Mg\sin\alpha - R_1 = M\left(\frac{1}{2}\right)$ **5**

$F_1 = R_1$

$R_1 = 8(16)^{\frac{3}{2}} = 512$

$\therefore F_1 = 8(16)^{\frac{3}{2}} = 512\ N$ **5**



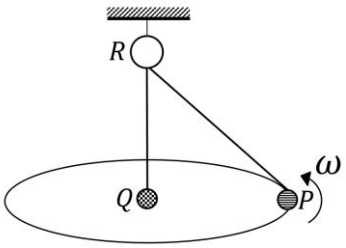
$P_1 = F_1 v_1$

$P_1 = 512 \cdot 16$

$P_1 = 8192\ W$ **5**

25

5. Two particles, P and Q of masses m and $2m$ respectively are attached to the two ends of a light inextensible string of length l which passes through a small smooth ring R, as shown in the figure. The particle P moves in a horizontal circle with a constant angular velocity ω while the string remains taut. Particle Q is at rest at the center of the circular path of P. Show that $\omega = \sqrt{\frac{3g}{l}}$. **Hence,** Find the radius of the circular path of P in terms of l.



$F = ma$

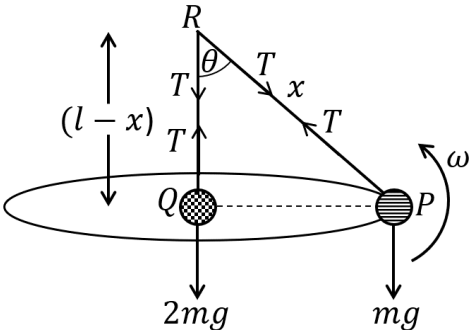
For $Q \uparrow$ $T = 2mg$ ———— $(\mathbf{1})$ **5**

For $P \uparrow$ $T\cos\theta = mg$ ———— $(\mathbf{2})$ **5**

$(\mathbf{2})/(\mathbf{1})$; $\cos\theta = \frac{1}{2} \Longrightarrow \theta = 60^0$

For $P \leftarrow$ $T\sin\theta = mx\sin\theta\,\omega^2$ —— $(\mathbf{3})$ **5**

$T = mx\omega^2$ ———— $(\mathbf{4})$



$cos\,\theta = \frac{l-x}{x} = \frac{1}{2}$

$2l - 2x = x \Longrightarrow x = \frac{2l}{3}$

Substitute in $(\mathbf{4})$;

$2mg = m\left(\frac{2l}{3}\right)\omega^2$

$\sqrt{\frac{3g}{l}} = \omega$ **5**

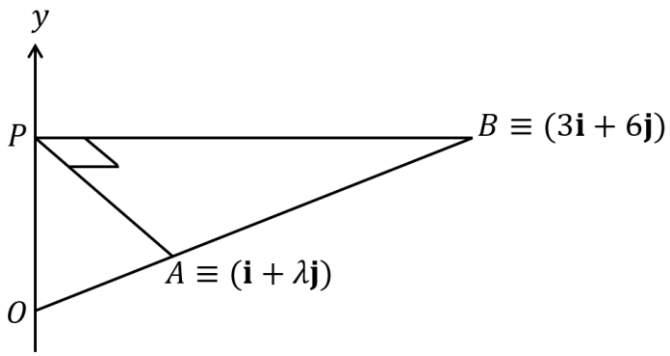
Radius $= x\,sin\,\theta$

$= \frac{2l}{3} sin\,60^0$

$= \frac{l}{\sqrt{3}}$ **5**

25

6. Let O, A and B are three collinear points and $\lambda \in \mathbb{R}$. In the usual notation, the position vectors of two points A and B, with respect to a fixed origin O, are $\mathbf{i}+\lambda\mathbf{j}$ and $3\mathbf{i} + 6\mathbf{j}$, respectively. Find the value of λ. Let P be the point on y-axis such that the lines AP and BP are perpendicular to each other. Find the two possible position vectors for P.



$\overrightarrow{OB} = k \cdot \overrightarrow{OA}$

$3\mathbf{i} + 6\mathbf{j} = k \cdot (\mathbf{i} + \lambda\mathbf{j})$

$k = 3, \ k\lambda = 6$

$\therefore \lambda = 2$ (5)

Let $\overrightarrow{OP} = \mu\mathbf{j}$

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = -\mathbf{i} + (\mu - 2)\mathbf{j}$

$\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP} = -3\mathbf{i} + (\mu - 6)\mathbf{j}$ (5)

$\overrightarrow{AP} \cdot \overrightarrow{BP} = (-\mathbf{i} + (\mu - 2)\mathbf{j}) \cdot (-3\mathbf{i} + (\mu - 6)\mathbf{j}) = 0 \quad (\because \overrightarrow{AP} \perp \overrightarrow{BP})$ (5)

$\mu^2 - 8\mu + 15 = 0$

$(\mu - 5)(\mu - 3) = \mathbf{0}$

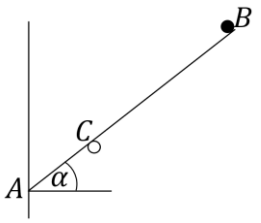
$\mu = 5$ or $\mu = 3$

(5) (5)

$\overrightarrow{OP} = 5\mathbf{j}$ and $\overrightarrow{OP} = 3\mathbf{j}$

25

7. A uniform rod ACB of length $4a$ and weight w is kept in equilibrium with the end A against a smooth vertical wall and end B attached to a particle of weight $2w$, by a smooth peg placed at C, such that $AC = a$. The rod makes an angle α with the horizontal, as shown in the figure, where $\tan\alpha = \sqrt{\frac{7}{3}}$. Find the magnitudes of the reactions at A and C in terms of w.



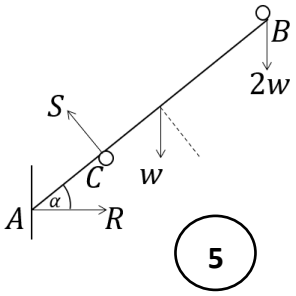
Resolving

$\nearrow\ R\cos\alpha - 3w\sin\alpha = 0$ **5**

$$R = 3w\tan\alpha$$

$$R = 3w\cdot\sqrt{\frac{7}{3}}$$

$$R = \sqrt{21}w$$ **5**



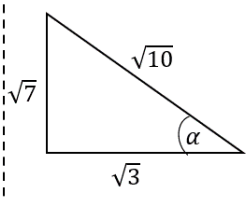
5

$\curvearrowleft A\quad S\cdot a - w\cdot 2a\cos\alpha - 2w\cdot 4a\cos\alpha = 0$ **5**

$$S = 10w\cos\alpha$$

$$S = 10w\cdot\sqrt{\frac{3}{10}}$$

$$S = \sqrt{30}w$$ **5**



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Aliter (To find S)

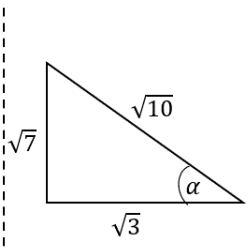
$\searrow\ S - R\sin\alpha - 3w\cos\alpha = 0$

$S - 3w\tan\alpha\sin\alpha - 3w\cos\alpha = 0$ **5**

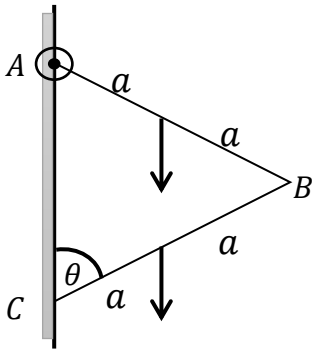
$$S = \frac{3w(\sin^2\alpha + \cos^2\alpha)}{\cos\alpha}$$

$$S = 3w\sec\alpha$$

$$S = \sqrt{30}w$$ **5**

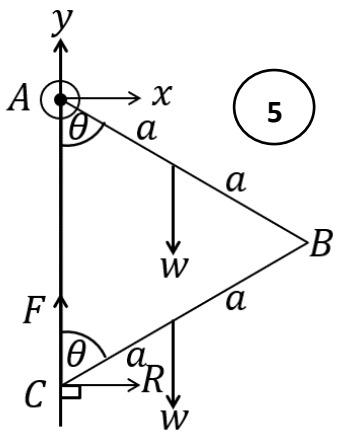


8. Two uniform rods AB, BC each of length $2a$ and weight W are smoothly jointed at B and the end A hinged to a rough vertical wall. The system rests in equilibrium in a vertical plane perpendicular to the wall with the end C of the rod BC inclined at an angle θ to the vertical. The coefficient of friction between the rod BC and the wall is μ . Show that $\tan\theta \geq \frac{2}{\mu}$.



System $\circlearrowleft A$ $\quad R \cdot 4a\cos\theta - 2w \cdot a\sin\theta = 0$

$$R = \frac{w\tan\theta}{2}$$ **5**



Rod BC $\circlearrowright$ B $\quad F \cdot 2a\sin\theta - R \cdot 2a\cos\theta - w \cdot a\sin\theta = 0$ **5**

$\div \sin\theta$

$$2F - 2R\cot\theta - w = 0$$

$$2F - 2\left(\frac{w\tan\theta}{2}\right)\cdot\frac{1}{\tan\theta} - w = 0$$ **5**

$$F = w$$

For equilibrium,

$$\mu \geq \frac{F}{R}$$

$$\mu \geq \frac{w}{\frac{w\tan\theta}{2}}$$ **5**

$$\tan\theta \geq \frac{2}{\mu}$$

25

9. Let A, B and C be three events of a sample space Ω. In the usual notation, it is given that $P(B) = \frac{3}{10}$ and $P(A \cup B) = \frac{3}{5}$. Find $P(A \cap B')$. If $A' \cap B$ and C are mutually exclusive and exhaustive events and $P(A) = \frac{1}{2}$, show that $P(C) = \frac{9}{10}$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(B) + \big(P(A) - P(A \cap B)\big)$

$\frac{3}{5} = \frac{3}{10} + P(A \cap B')$ **5**

$\therefore P(A \cap B') = \frac{3}{10}$ **5**

Since $A' \cap B$ and C are mutually exclusive and exhaustive events,

$P(A' \cap B) + P(C) = 1$ **5**

$\frac{1}{10} + P(C) = 1$

$\therefore P(C) = \frac{9}{10}$

$P(A' \cap B) = P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{1}{2} + \frac{3}{10} - \frac{3}{5} = \frac{1}{5}$ **5**

$\therefore P(A' \cap B) = \frac{3}{10} - \frac{1}{5} = \frac{1}{10}$ **5**

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10. The mean of a set of five observations each of which is a positive integer is 4. The unique mode of the set is 3 and the range of the observations is 5. **Given that** the median is 3, find two possible sets of observations.

Let the observations in ascending order be $a \le b \le c \le d \le e$.

Since Mean = 4; $\left(\frac{a+b+c+d+e}{5}\right) = 4 \Rightarrow a+b+c+d+e = 20$ **5**

Since there are five observations and the median is 3; $c = 3$ **5**

Since 3 is the unique mode;

Case I: 3 occurs exactly twice $a,\ 3,\ 3,\ d,\ e$

Since the range is 5; $(e - a) = 5$ **5**

{2,3,3,5,7} **5**

Case II: 3 occurs four times $3,\ 3,\ 3,\ 3,\ e$

Since the range is 5; $(e - 3) = 5 \Rightarrow e = 8$

{3,3,3,3,8} **5**

25

PART B

11. (a) A motor car X, moving along a straight road, passes a point A with velocity u and a uniform acceleration f. It accelerates until its velocity becomes λu, where $(\lambda > 2)$. At a point *B*, it begins to move with uniform deceleration f and reaches point C when its velocity is $2u$. Another motor car Y, moving along the same road in the same direction with uniform velocity of $2u$, passes the point A a time $\frac{u}{f}$ before motor car X passes same point A.

Sketch velocity-time graphs for the motions of both X and Y in the same diagram. **Hence,**

(i) show that $AB = \frac{1}{2f}u^2(\lambda^2 - 1)$

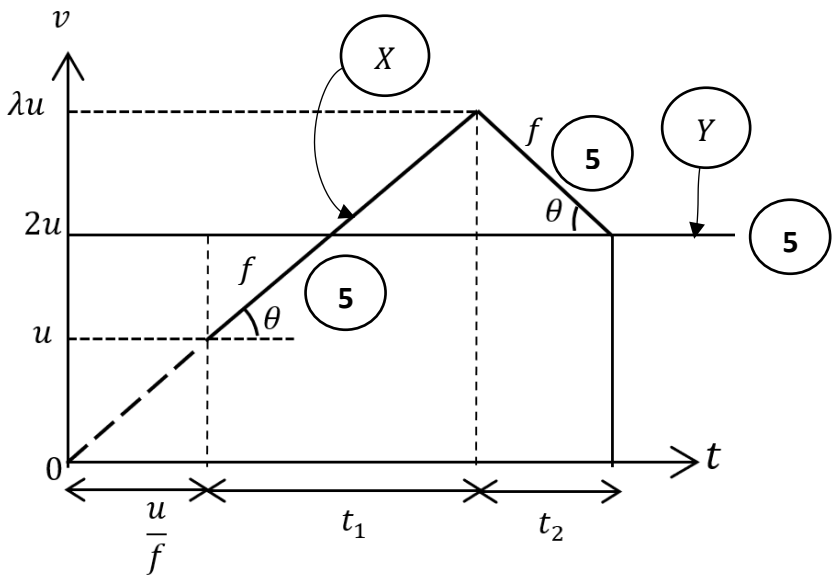
(ii) show that the total distance travelled by *X*, when it reaches the point *C* is $\frac{1}{2f}u^2(2\lambda^2 - 5)$

(iii) show that if $\lambda \leq 2 + \frac{\sqrt{10}}{2}$, *X* cannot overtake *Y*.

(b) A ship S is sailing with a constant velocity of v kmh^{-1} relative to the Earth in a direction making an angle α west of north. A warship W is sailing due west with a constant velocity of $u\ (< v\sin\alpha)$kmh^{-1} relative to the Earth. The warship detects ship S as an enemy vessel and attempts to intercept it. At a certain instant, the ship is directly south-west of the warship at a distance d km. By sketching the velocity triangle, determine the path of the ship relative to the warship. **Hence,** find the shortest distance l between the warship and the ship. If the maximum firing range of the guns fixed to the warship is $R\ (> l)$km, show that the ship remains within the firing range of

the guns for a time interval of $\frac{2\sqrt{R^2 - l^2}}{\sqrt{v^2 + u^2 - 2uv\sin\alpha}}$ hours.

(a)



15

(i) $AB = \dfrac{(\lambda u + u)}{2} \cdot t_1$ **5**

$= \dfrac{(\lambda u + u)}{2} \cdot \dfrac{(\lambda u - u)}{f}$ **5**

$= \dfrac{1}{2f} u^2(\lambda^2 - 1)$

from graph

$f = \tan\theta = \dfrac{(\lambda u - u)}{t_1}$ **5**

$t_1 = \dfrac{(\lambda u - u)}{f}$

15

(ii) $AC = AB + BC$

$AC = \dfrac{1}{2f} u^2(\lambda^2 - 1) + \dfrac{1}{2f} u^2(\lambda^2 - 4)$ **5**

$AC = \dfrac{1}{2f} u^2(2\lambda^2 - 5)$

$f = \tan\theta = \dfrac{(\lambda u - 2u)}{t_2}$ **5**

$t_2 = \dfrac{(\lambda u - 2u)}{f}$

$BC = \dfrac{(\lambda u + 2u)}{2} \cdot t_2$

$= \dfrac{(\lambda u + 2u)}{2} \cdot \dfrac{(\lambda u - 2u)}{f}$

$= \dfrac{\lambda^2 u^2 - 4u^2}{2f}$

$= \dfrac{u^2(\lambda^2 - 4)}{2f}$ **5**

15

(iii) X cannot overtake Y if, for any given time interval, the total distance travelled by X is less than or equal to the total distance travelled by Y.

Distance travelled by $X \leq$ Distance travelled by Y from A to C. **5**

5 **5**

$\dfrac{1}{2f} u^2(2\lambda^2 - 5) \leq 2u \cdot \left(\dfrac{u}{f}\right) + 2u \cdot (t_1 + t_2)$

$\dfrac{1}{2f} u^2(2\lambda^2 - 5) \leq 2u \cdot \left(\dfrac{u}{f} + \dfrac{(\lambda u - u)}{f} + \dfrac{(\lambda u - 2u)}{f}\right)$

$\dfrac{u^2}{2f}(2\lambda^2 - 5) \leq \dfrac{2u^2}{f} \cdot (1 + \lambda - 1 + \lambda - 2)$

$(2\lambda^2 - 5) \leq 8 \cdot (\lambda - 1)$ **5**

$2\lambda^2 - 8\lambda + 3 \leq 0$

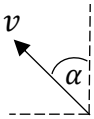
$$\lambda^2 - 4\lambda + \frac{3}{2} \leq 0$$

$$(\lambda - 2)^2 \leq \frac{5}{2} \quad \textbf{5}$$

$$\lambda \leq 2 \pm \sqrt{\frac{5}{2}}$$

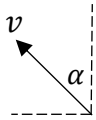
$$\lambda \leq 2 + \frac{\sqrt{10}}{2} \quad (\because \lambda > 2 \; given) \quad \textbf{5}$$

30

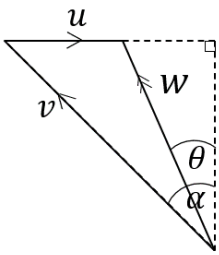
(b) $(S, E) =$ 

, $(W, E) = \leftarrow u$

$$\mathbf{V}(S, W) = \mathbf{V}(S, E) + \mathbf{V}(E, W) \quad \textbf{5}$$

$=$ 

$+ \rightarrow u$ **5**



15

Let w be the velocity of the warship relative to ship

$$w \sin\theta = v \sin\alpha - u \quad \text{——— } (\mathbf{1})$$

$$w \cos\theta = v \cos\alpha \quad \text{——— } (\mathbf{2}) \quad \textbf{5}$$

$(\mathbf{1})^2 + (\mathbf{2})^2;\ w^2 = v^2 + u^2 - 2uv \sin\alpha$

$$w = \sqrt{u^2 + v^2 - 2uv \sin\alpha} \quad \textbf{5}$$

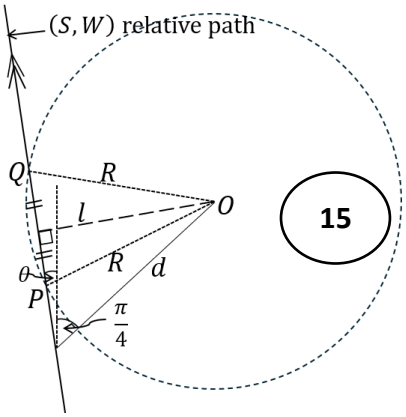
$\frac{(\mathbf{1})}{(\mathbf{2})}$; $\tan\theta = \frac{(v \sin\alpha - u)}{v \cos\alpha}$

Shortest distance $(l) = d \cdot \sin\left(\theta + \frac{\pi}{4}\right)$ **5**

$$= \frac{d}{\sqrt{2}} \cdot (\sin\theta + \cos\theta) \quad \textbf{5}$$

$$= \frac{d}{\sqrt{2}w} \cdot \left((v \sin\alpha - u) + v \cos\alpha\right)$$

$$= \frac{d}{\sqrt{2}} \cdot \frac{[v(\sin\alpha + \cos\alpha) - u]}{\sqrt{u^2 + v^2 - 2uv \sin\alpha}} \quad \textbf{5}$$



15

Let the time in which the ship remains within the firing range of the gun is t

By applying $S = ut$

$$t = \frac{PQ}{|\mathbf{V}(S,W)|}$$

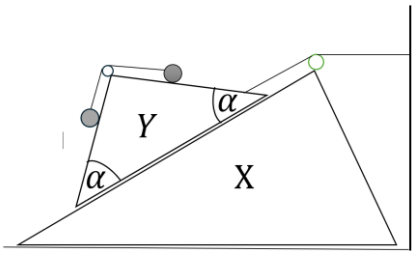
$$= \frac{2\sqrt{R^2 - l^2}}{|\mathbf{V}(S,W)|}$$

$$= \frac{2\sqrt{R^2 - l^2}}{w} \quad \textbf{5}$$

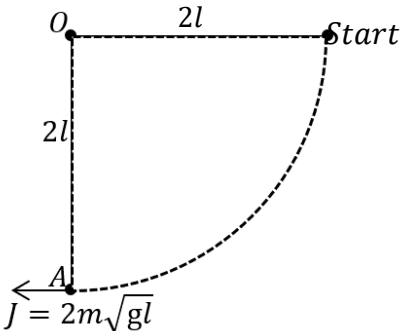
$$= \frac{2\sqrt{R^2 - l^2}}{\sqrt{u^2 + v^2 - 2uv\sin\alpha}} \text{ hours} \quad \textbf{5}$$

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12. (a) The vertical cross-section through the centres of mass of two smooth uniform wedges, X and Y, particles P and Q is shown in the figure. The smooth wedge X, of Mass M is placed on a smooth horizontal table. On its inclined face, which makes an angle $\alpha \left(< \frac{\pi}{4}\right)$ with the horizontal, another smooth wedge Y, of mass $2m$, whose inclined faces make an angle α is placed. Two particles P and Q, each of mass m, are attached to the two ends of a light inextensible string which passes over a fixed smooth pulley at a vertex of wedge Y. One end of another light inextensible string, which passes over a fixed smooth pulley at a vertex of wedge X, is attached to wedge Y, and the other end is fixed to a point on a vertical wall in the same plane, such that the string remains horizontal. The system is released from rest with the strings taut. Write down equations sufficient to determine the tensions in the strings.

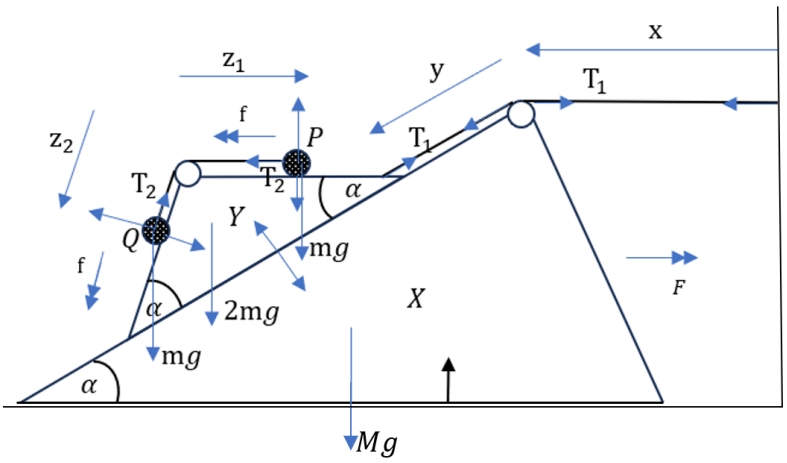


(b) As shown in the figure two ends of a light inextensible string of length $2l$ is attached to a small particle P of mass m and fixed-point O. The particle is released from rest at the same horizontal level of O with the string taut. In the subsequent motion particle P moves in a vertical circular path and reaches the lowest point A and then receives a horizontal impulse of magnitude $2m\sqrt{gl}$ in the direction of its motion. Find the velocity immediately after the impulse.



Show that the velocity and the tension in the string when OP make an angle 60^0 with the downward vertical are $\sqrt{14gl}$ and $\frac{15mg}{2}$ respectively. If at the same moment the string is suddenly cut without imparting any additional impulse and then the particle moves freely under the gravity and reaches point B which is at the same horizontal level of point A, show that the time taken by particle P to reach B is $\left(\frac{\sqrt{42}+5\sqrt{2}}{2}\right)\sqrt{\frac{l}{g}}$

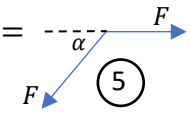
(a) Forces (5)

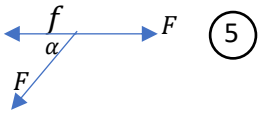


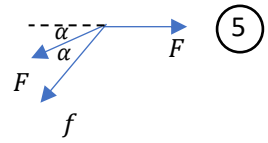
$x + y = k$; k is a constant
$\ddot{x} + \ddot{y} = 0$
$\ddot{x} = -\ddot{y}$ (5)

$z_1 + z_2 = k_1$; k_1 is a constant
$\ddot{z_1} + \ddot{z_2} = 0$
$\ddot{z_1} = -\ddot{z_2}$

$a_{(X,E)} = F \longrightarrow$ (5)

$a_{(Y,E)} =$  (5)

$a_{(P,E)} =$  (5)

$a_{(Q,E)} =$  (5)

$\underline{F} = m\underline{a}$

For P; $\leftarrow$ $T_2 = m(f - F + Fcos\alpha)$ (10)

For Q;  $mg\ sin2\alpha - T_2 = m(f + F\ cos\alpha - F\ cos2\alpha)$ (10)

For the system P, Q and Y 

$4mg\ sin\alpha - T_1 = m(F - F\ cos\alpha + f\ cos\alpha) + m(F - F\ cos\alpha + f\ cos\alpha) + 2m(F - Fcos\alpha)$ (15)

For the system P, Q, X, Y $\longrightarrow$

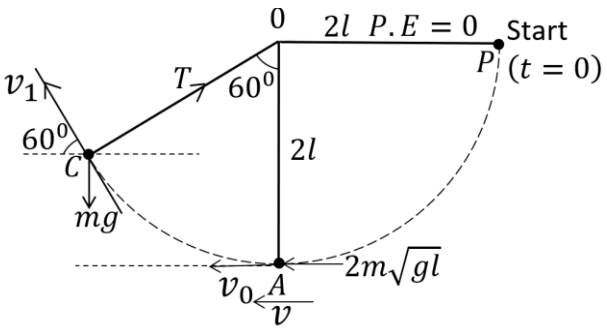
$T_1 = m(F - f - F\ cos\alpha) + m(F - F\ cos\alpha + f\ cos2\alpha) + 2m(F - Fcos\alpha) + MF$ (20)

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(b) By conservation of energy (Start to A)

$0 = \frac{1}{2} \cdot mv^2 - mg(2l)$ (5)

$v = 2\sqrt{gl}$ (5)



$\leftarrow$ At point A

$\underline{I} = \Delta m\underline{v}$

$2m\sqrt{gl} = m(v_0 - v)$ (5)

$2m\sqrt{gl} = m(v_0 - 2\sqrt{gl})$

$v_0 = 4\sqrt{gl}$ (5)

By conservation of energy (A to C)

$\frac{1}{2} \cdot m(v_0)^2 - mg \cdot 2l = \frac{1}{2} \cdot mv_1{}^2 - mg(2l\ sin\ 30^0)$ (10)

$\frac{1}{2} \cdot m\left(4\sqrt{gl}\right)^2 - mg \cdot 2l = \frac{1}{2} \cdot mv_1{}^2 - mg\left(2l \cdot \frac{1}{2}\right)$

$v_1 = \sqrt{14gl}$ (5)

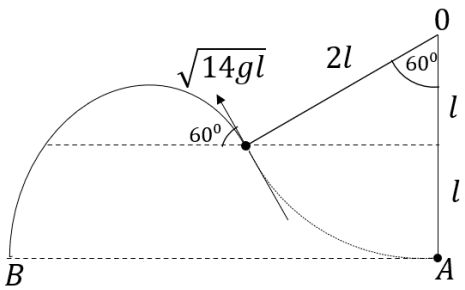
At C, $\nearrow$ $\underline{F} = m\underline{a}$

$$T - mg\cos 60^0 = m\left(\frac{{v_1}^2}{2l}\right) \quad (10)$$

$$T - mg\cdot\frac{1}{2} = m\left(\frac{14gl}{2l}\right)$$

$$T = \frac{15mg}{2} \quad (5)$$

For the particle P when move freely under the gravity,



$$\uparrow S = ut + \frac{1}{2}at^2$$

$$-l = v_1 \sin 60^0\, t + \frac{1}{2}(-g)t^2 \quad (5)$$

$$-l = \sqrt{14gl}\cdot\frac{\sqrt{3}}{2}t - \frac{1}{2}gt^2$$

$$gt^2 - \sqrt{42gl}\,t - 2l = 0 \quad (5)$$

$$t = \frac{\sqrt{42gl} \pm \sqrt{42gl - 4g(-2l)}}{2\cdot g} \quad (5)$$

Since $t > 0$,

$$t = \frac{\sqrt{42} \pm \sqrt{5\sqrt{2}}}{2}\cdot\left(\frac{l}{g}\right)$$

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13. A small cubical block of mass $8m$ is attached to one end A of a light elastic spring AB of natural length d and modulus of elasticity $6mg$. The spring and block are at rest on a horizontal table with AB equal to d and lying perpendicular to the face to which A is attached. Another block of equal physical dimensions, but of mass m, moving with a speed $\sqrt{2ga}$ in the direction parallel to BA impinges on the free end B of the spring.

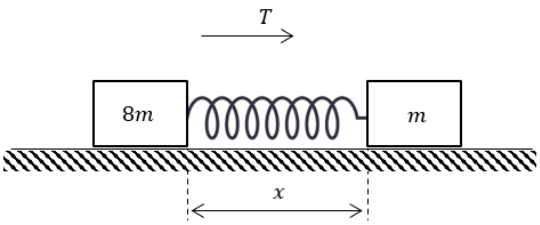
It is assumed that the contact between the lighter block and the table is smooth.

Assuming that the heavier block is held fixed, AB remains straight and horizontal in the subsequent motion, when the one end of the spring B is at a distance x from A, Show that the equation of the motion of the lighter block is given by $\ddot{X} = -\omega^2 X$, where $X = x - d$

By assuming solutions for x in the form $x = d + h\cos\omega t + k\sin\omega t$, find the values of the constants h and k. **Hence,** Find the minimum length of the spring and **deduce** that $d > \frac{a}{3}$.

If $d = a$, show that the ratio between the time taken to compress the length of the spring to half of its length for the first time and time taken to reach the minimum length of the spring for the first time is 2: 3.

Now, the heavier block is free to move on the table and the coefficient of friction between the heavier block and the table is μ, find the least value of μ in terms of a and d in order to keep the heavier block not moving.



Applying **g** $\underline{F} = m\underline{a}$

$T = m\ddot{x}$

$\frac{6mg(d-x)}{d} = m\ddot{x}$ (5) ;

$\ddot{x} + \frac{6g}{d}(x-d) = 0$ (5)

$\ddot{X} + \frac{6g}{d}X = 0$

$\ddot{x} = -\frac{6g}{d}X$

$\ddot{x} = -\omega^2 X$ (5)

By hook law,

$T = \frac{6mg(d-x)}{d}$ (5)

$X = x - d$

$\therefore \dot{X} = \dot{x}$

$\ddot{X} = \ddot{x}$

where $\omega = \sqrt{\frac{6g}{d}}$ and $X = x - d$ (5)

25

when $t = 0$,

$x = d, \qquad \dot{x} = \sqrt{2ga}$ (5)

$x = d + h\,cos(\omega t) + k\,sin(\omega t) \Rightarrow d = d + h\,cos(0) + k\,sin(0)$ (5)

$\therefore h = 0$ (5)

$\dot{x} = -h\omega \cdot \sin(\omega t) + k\omega \cdot \cos(\omega t) \Rightarrow \sqrt{2ga} = -h\omega \cdot \sin(0) + k\omega \cdot \cos(0)$ (5)

$$k = \frac{\sqrt{2ga}}{\omega} \quad (5)$$

$$k = \sqrt{\frac{2ga}{\frac{6g}{d}}}$$

$$k = \sqrt{\frac{da}{3}} \quad (5)$$

$x = d + h\,cos(\omega t) + k\,sin(\omega t)$

$x = d + (0)\,cos(\omega t) + k\,sin(\omega t) \Rightarrow x = d + k\,sin(\omega t)$ (5)

Since $-1 \leq sin(\omega t) \leq 1$

$\therefore (x)_{min} = d + k(-1)$

$(x)_{min} = d - k$ (5)

$(x)_{min} > 0$ (5)

$d > k$

$d > \sqrt{\frac{da}{3}}$ (5)

$d > \frac{a}{3}$ **50**

If $d = a \Rightarrow \quad (x)_{min} = a - \sqrt{\frac{da}{3}}sin(\omega t) = a - \frac{a}{\sqrt{3}}sin(\omega t)$, where $\omega = \sqrt{\frac{6g}{a}}$

If the spring compressed to half of its length when $t = t_1$

(5) $\frac{a}{2} = a - \frac{a}{\sqrt{3}}sin(\omega t_1)$ (5)

$sin(\omega t_1) = \frac{\sqrt{3}}{2}$ (5)

$\omega t_1 = \frac{\pi}{3} \Rightarrow t_1 = \frac{\pi}{3\omega}$ (5)

If the length of the spring is minimum when $t = t_2$

$(x)_{min} = a - \frac{a}{\sqrt{3}}.1$ (5)

When $sin(\omega t_2) = 1$ (5)

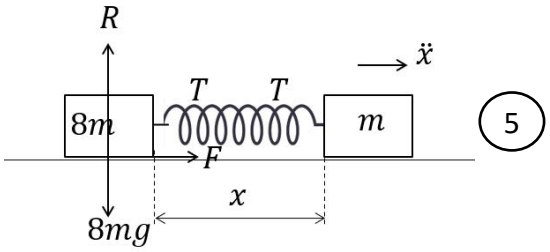
$\omega t_2 = \frac{\pi}{2} \Rightarrow t_2 = \frac{\pi}{2\omega}$ (5)

$\therefore t_1 : t_2 = \frac{\pi}{3\omega} : \frac{\pi}{2\omega}$ (5)

$= 2 : 3$

40

Assuming that $8m$ block is not moving,



(5)

For $8m$; $\uparrow \underline{F} = m\underline{a}$ $\rightarrow \underline{F} = m\underline{a}$

$R - 8mg = 0$ $F - T = 0$

$R = 8mg$ (5) $F = T = \frac{6mg(d-x)}{d}$ (5)

To avoid sliding, $\mu \geq \frac{|F|}{R}$ (5)

$$\mu \geq \frac{\left\lfloor \frac{6mg(d-x)}{d} \right\rfloor}{8mg}$$

$$\mu \geq \left(\frac{3(d-x)}{4d}\right)_{max}$$ (5)

$$\mu \geq \frac{3}{4d}.(d - x_{min})$$ (5)

$$\mu \geq \frac{3}{4d}.\big(d - (d-k)\big)$$

$$\mu \geq \frac{3}{4d}.k$$

$$\mu \geq \frac{3}{4d}.\sqrt{\frac{da}{3}}$$

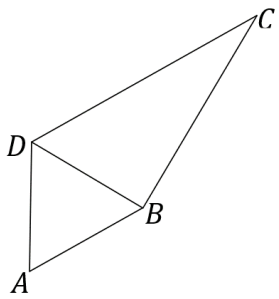
$$\mu \geq \frac{1}{4}.\sqrt{\frac{3a}{d}}$$ (5)

35

14. (a) Let $OACB$ be a parallelogram with O as the vector origin and **a** and **b** be two unit **vectors**. The position vectors of the vertices A and B with respect to an origin O, are $\alpha\mathbf{a}$ and $\beta\mathbf{b}$ respectively. The point P is on OA such that $OP : OA = \alpha : 1$ and the point Q is on AC such that $AQ : AC = \beta : 1$
Using the triangular law of vector addition, show that $\overrightarrow{PC} = \alpha(1-\alpha)\mathbf{a} + \beta\mathbf{b}$ and $\overrightarrow{BQ} = \alpha\mathbf{a} - \beta(1-\beta)\ \mathbf{b}$

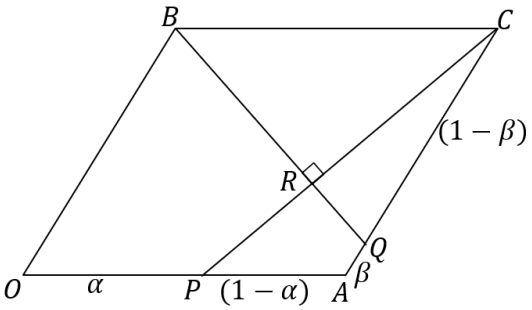
The lines PC and BQ meet at R such that $B\hat{R}P = \frac{\pi}{2}$. Show that the angle θ, between the unit vector **a** and **b** is given by $\cos^{-1}\left(\frac{\beta^2(1-\beta) - \alpha^2(1-\alpha)}{\alpha\beta(\alpha+\beta-\alpha\beta)}\right)$.

(b) A quadrilateral $ABCD$ is shown in the figure, where AD is vertical. Triangle ABD is equilateral with side length $2a$, $C\hat{B}D = 90°$ and $CD = 4a$. Forces of magnitudes $6P, 4P, \alpha P, \beta P$ and $8\sqrt{3}P$ act along AB, AD, DC, DB and CB respectively in the directions indicated by the order of the letters where α and β are real constants.



Given that the resultant of the system is parallel to DB in the sense from D to B. Show that $\alpha = 6$.
If the magnitude of the resultant force is $6P$, find the possible value for β.
Calculate the moment of the couple that must be added to the system so that it reduces to a single force along DB.
The resultant system is brought into equilibrium by applying a force F_1 at A, perpendicular to BD, and a force F_2 at C. Find the values of F_1 and F_2 .

(a)



$\frac{PA}{OA} = \frac{1-\alpha}{1}$ $\qquad$ $\frac{CQ}{CA} = \frac{1-\beta}{1}$

$\therefore \overrightarrow{PA} = (1-\alpha)\,\overrightarrow{OA}$ $\qquad$ $\therefore \overrightarrow{CQ} = (1-\beta)\,\overrightarrow{CA}$

$= (1-\alpha)\,\alpha\,\underline{a}$ $\qquad$ $= (1-\beta)\,(-\beta\underline{b})$

$\overrightarrow{PC} = \overrightarrow{PA} + \overrightarrow{AC}$ (5) $\qquad$ $\overrightarrow{BQ} = \overrightarrow{BC} + \overrightarrow{CQ}$ (5)

(5) (5) $\qquad$ (5) (5)

$= (1-\alpha)\,\alpha\,\underline{a} + \beta\,\underline{b}$ $\qquad$ $= \alpha\,\underline{a} + (1-\beta)\,(-\beta\underline{b})$

$= \alpha(1-\alpha)\,\underline{a} + \beta\,\underline{b}$ $\qquad$ $= \alpha\,\underline{a} - \beta(\beta-1)\,\underline{b}$

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$$\overrightarrow{PC}\,.\overrightarrow{BQ} = \Big((1-\alpha)\,\alpha\,\underline{a} + \beta\,\underline{b}\Big)\cdot\Big(\alpha\,\underline{a} + (1-\beta)\,(-\beta\underline{b})\Big)$$
$$= \alpha^2(1-\alpha)\,\underline{a}\cdot\underline{a} - \beta^2(1-\beta)\,\underline{b}\cdot\underline{b} + -\alpha\beta(1-\alpha)(1-\beta)\,\underline{a}\cdot\underline{b} + \alpha\beta\,\underline{b}\cdot\underline{a}$$
$$= \alpha^2(1-\alpha)\,\underline{a}\cdot\underline{a} - \beta^2(1-\beta)\,\underline{b}\cdot\underline{b} + -\alpha\beta\big((1-\alpha)(1-\beta)-1\big)\,\underline{a}\cdot\underline{b}$$
(5) (5) (5)
$$= \alpha^2(1-\alpha)\,\underline{a}\cdot\underline{a} - \beta^2(1-\beta)\,\underline{b}\cdot\underline{b} + \alpha\beta(\alpha+\beta-\alpha\beta)\,\underline{a}\cdot\underline{b}$$

$PC \perp BQ \Longrightarrow \overrightarrow{PC}\,.\,\overrightarrow{BQ} = O$ (5)

$\therefore O = \alpha^2(1-\alpha)\,\underline{a}\cdot\underline{a} - \beta^2(1-\beta)\,\underline{b}\cdot\underline{b} + \alpha\beta(\alpha+\beta-\alpha\beta)\,\underline{a}\cdot\underline{b}$ (5)

As $\underline{a}$ & $\underline{b}$ are unit vectors $|a| = |b| = 1$
$\therefore \underline{a}\cdot\underline{a} = 1$ and $\underline{b}\cdot\underline{b} = 1$ (5)

If the angle between $\underline{a}$ and $\underline{b}$ is θ
$a\cdot b = |a||b|\cos\theta \Longrightarrow a\cdot b = \cos\theta$ (5)

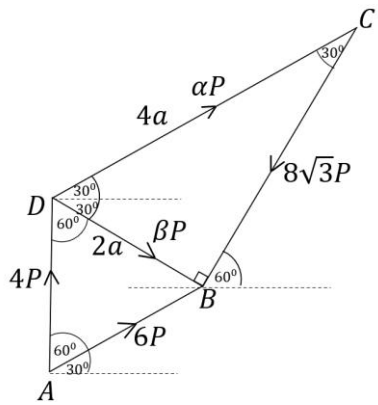
$O = \alpha^2(1-\alpha) - \beta^2(1-\beta) + \alpha\beta(\alpha+\beta-\alpha\beta)\,Cos\,\theta$

$\therefore Cos\,\theta = \dfrac{\beta^2(1-\beta) - \alpha^2(1-\alpha)}{\alpha\beta(\alpha+\beta-\alpha\beta)}$ (5)

$\therefore \theta = Cos^{-1}\dfrac{\beta^2(1-\beta) - \alpha^2(1-\alpha)}{\alpha\beta(\alpha+\beta-\alpha\beta)}$

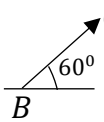
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(b)



Since the resultant in the direction $\overrightarrow{DB}$,

The sum of the components of forces $\perp$ to $DB = 0$.



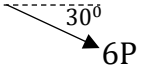
$-8\sqrt{3}\text{P} + \alpha\text{P}\cos 30^0 + 6\text{P}\cos 30^0 + 4\text{P}\cos 30^0 = 0$ (10)

$$-8\sqrt{3}\text{P} + \alpha\text{P}\cdot\frac{\sqrt{3}}{2} + 6\text{P}\cdot\frac{\sqrt{3}}{2} + 4\text{P}\cdot\frac{\sqrt{3}}{2} = 0$$ (5)

$\alpha = 6$

15

Since Resultant force =

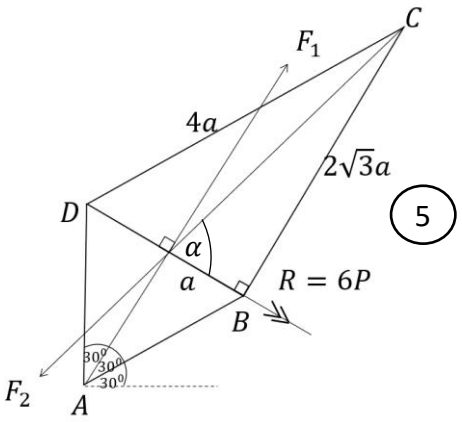


$\rightarrow 6\text{P}\cos 30^0 = 6\text{P}\cos 30^0 + \alpha\text{P}\cos 30^0 - 8\sqrt{3}\text{P}\cos 60^0 + \beta P\cos 30^0$ (10)

$$6\text{P}\cdot\frac{\sqrt{3}}{2} = 6\text{P}\cdot\frac{\sqrt{3}}{2} + 6\text{P}\cdot\frac{\sqrt{3}}{2} - 8\sqrt{3}\text{P}\cdot\frac{1}{2} + \beta P\cdot\frac{\sqrt{3}}{2}$$

$\beta = 2$ (5)

15



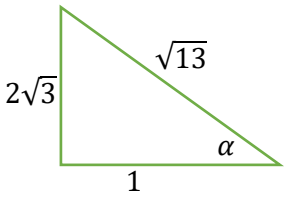
5

$$\tan\alpha = \frac{2\sqrt{3}a}{a}$$

$$\tan\alpha = 2\sqrt{3}$$

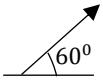
$$\cos\alpha = \frac{1}{\sqrt{13}}$$

5



$$\sin\alpha = \frac{2\sqrt{13}}{\sqrt{13}} = 2$$

Since the system is in equilibrium,



$$F_1 - F_2 \sin\alpha = 0$$

5



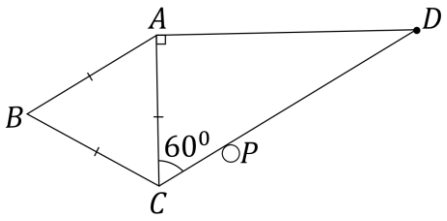
$$F_2 \cos\alpha - 6P = 0$$

$$F_2 = 6\sqrt{13}P$$ 5

$$F_1 = 12\sqrt{3}P$$ 5

25

15.(a) Uniform rods AB and BC each of length $2a$ and weight W and two other uniform rods AD and CD of weights W and $2W$ and of lengths $2\sqrt{3}a$ and $4a$ respectively are freely jointed at their ends. A light rod AC of length $2a$ is freely jointed to A and C. The end D is smoothly hinged to a fixed point and the rod CD is in contact with a small smooth peg P where $PD = 3a$. The system is in equilibrium in a vertical plane as shown in the figure with AD horizontal.

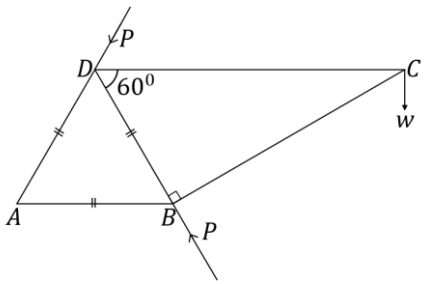


(i) Find the reaction at B on the rod AB

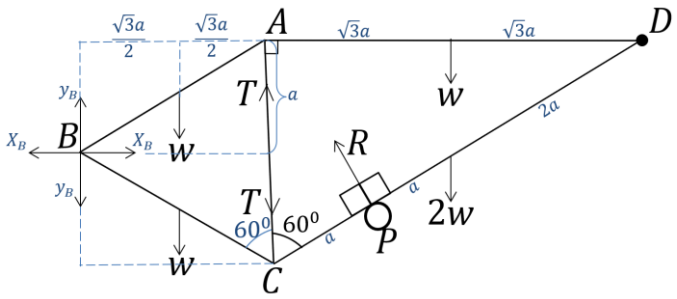
(ii) Show that the thrust in the light rod is $\frac{3W}{2}$

Also, find the reaction exerted on the rod CD by the peg P

(b) The frame work shown in the figure consists of five light rods AB, BC, CD, DA and DB that are smoothly jointed at their ends. It is given that $AB = AD = BD = a$, $D\hat{B}C = 90^0$, $B\hat{D}C = 60^0$. A load w is suspended at the joint C and the frame work is smoothly hinged to a fixed point at A and kept in equilibrium in a vertical plane with AB and DC horizontal, by two forces each of magnitude P applied in $\overrightarrow{BD}$ and $\overrightarrow{DA}$ directions to it at the joints B and D respectively. Find the value of P. Draw a stress diagram using Bow's notation. **Hence,** find the stresses in the rods stating whether they are tensions or thrusts.



(a)



For the equilibrium of,

Rod AB; $A\circlearrowleft$ $w \cdot a\sin 60^0 + X_B \cdot 2a\cos 60^0 - Y_B \cdot 2a\cos 30^0 = 0$ (10)

$$\frac{\sqrt{3}}{2}w + X_B - \sqrt{3}\,Y_B = 0 \text{ ——————— } (\mathbf{1})$$

Rod BC; $C\circlearrowleft$ $w \cdot a\sin 60^0 + X_B \cdot 2a\cos 60^0 + Y_B \cdot 2a\sin 60^0 = 0$ (10)

$$\frac{\sqrt{3}}{2}w + X_B + \sqrt{3}\,Y_B = 0 \text{ ——————— } (\mathbf{2})$$

$(\mathbf{1}) + (\mathbf{2})$; $\quad X_B = -\frac{\sqrt{3}}{2}w$ (5)

$(\mathbf{2}) - (\mathbf{1})$; $\quad Y_B = 0$ (5)

∴ Reaction on AB at B is $\leftarrow \frac{\sqrt{3}}{2}w$ (5)

Rod DA and AB; D ↺

$w \cdot \sqrt{3}a - T \cdot 2\sqrt{3}\,a + w \cdot \left(2\sqrt{3}a + a\cos 30^0\right) + X_B \cdot 2a\cos 60^0 = 0$ (10)

$$\sqrt{3}w - 2\sqrt{3}T + \frac{5\sqrt{3}}{2}w + \left(-\frac{\sqrt{3}}{2}w\right) \cdot 2 \cdot \frac{1}{2} = 0$$

$$T = \frac{3w}{2}$$

System; D ↺ $3w \cdot \sqrt{3}a + 2w \cdot \left(2\sqrt{3}a + a\cos 30^0\right) - R \cdot 3a = 0$ (10)

$$3w \cdot \sqrt{3}a + 2w \cdot \left(2\sqrt{3}a + a\frac{\sqrt{3}}{2}\right) - R \cdot 3a = 0$$

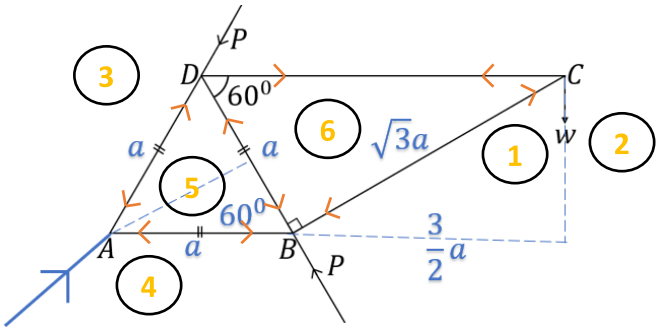
$$R = \frac{8}{\sqrt{3}}w \quad (5)$$

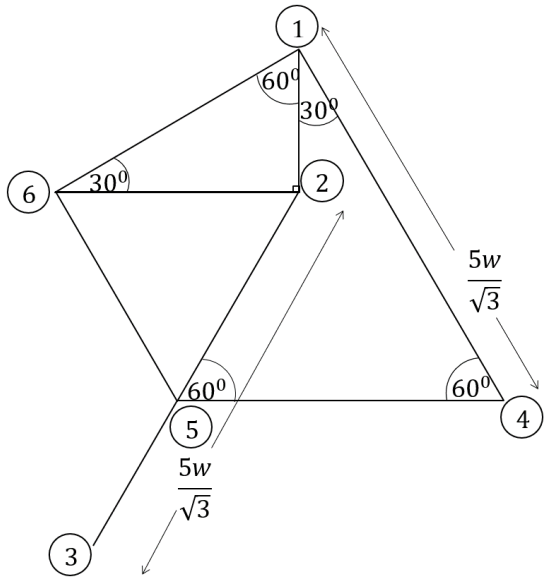
60

(b) For the equilibrium of the frame,

A ↺ $P \cdot a\cos 30^0 - w \cdot \left(a + \frac{3}{2}a\right) = 0$ (5)

$$P = \frac{5w}{\sqrt{3}} \quad (5)$$





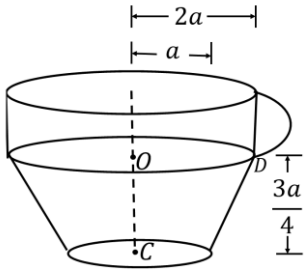
3 joints B, C, D 10x3 (30)

Rod	Tension	Thrust	Magnitude
AB (4,5)		✓ (5)	$\frac{4w}{\sqrt{3}}$ (5)
BC (1,6)		✓ (5)	$2w$ (5)
CD (2,6)	✓ (5)		$\sqrt{3}w$ (5)
DA (3,5)		✓ (5)	$\sqrt{3}w$ (5)
DB (5,6)		✓ (5)	$\frac{2w}{\sqrt{3}}$ (5)

90

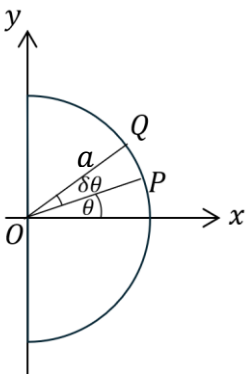
16. Show that the centre of mass of a thin uniform wire in the shape of a semicircular arc of radius a is at a distance $\frac{2a}{\pi}$ from its centre and that the centre of mass of a uniform hollow right circular cone of height h is at a distance $\frac{h}{3}$ from the centre of the base of the cone.

A vessel is formed by rigidly fixing a uniform thin shell in the shape of a frustum of hollow right circular cone, whose upper and lower circular rims have radii $2a$ and a respectively and height $\frac{3a}{4}$, to a thin cylindrical shell of radius $2a$ and height $\frac{a}{2}$ at the upper rim of the frustum and to a thin circular plate of radius a at the lower rim, as shown in the figure. A handle made of a thin semicircular wire of radius is fixed to the cylindrical part.



The mass per unit area of the frustum, the cylinder and the plate is ρ and the mass per unit length of the wire is $4a\rho$. Show that the centre of mass of the vessel is located at $\left(\frac{5a}{31}, \frac{2a}{31\pi}(4\pi+1)\right)$ on the plane of symmetry Oxy where the $\boldsymbol{x}$ axis lies along OC and the $\boldsymbol{y}$ axis lies along OD of which the origin is O.

It is suggested that a thin wire ring of radius $2a$ should be attached along the rim of the cylindrical shell such that OC remains horizontal when the vessel is suspended from handled by a smooth peg. Find the mass per unit length of the wire ring used.



According to the symmetry, C.O.G lies on the $x-axis$ (5)

Mass per unit length is ρ.

Length of $PQ = a \cdot \delta\theta$

$\therefore \delta x = (a\delta\theta) \cdot \rho$

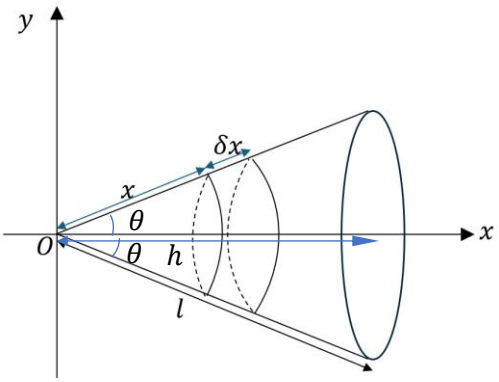
$x = r\cos\theta$

$$\overline{x} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\cos\theta \cdot a\,\rho\, d\theta}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\,\rho\, d\theta} \quad (5) \quad (5)$$

$$\overline{x} = \frac{a^2\rho \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta}{a\rho \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta}$$

$$\overline{x} = \frac{a^2\rho[\sin\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}{a\rho[\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}} \quad (5) = \frac{a\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right)}{\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)} = \frac{a\left(1+\sin\frac{\pi}{2}\right)}{\pi} \quad (5)$$

$$= \frac{2a}{\pi}$$



According to the symmetry C.O.G. lies on the $x-axis$. (5)
Mass pert unit area is ρ

$h = l\cos\theta$
$\delta m = 2\pi(x\sin\theta)\delta x \cdot \rho$

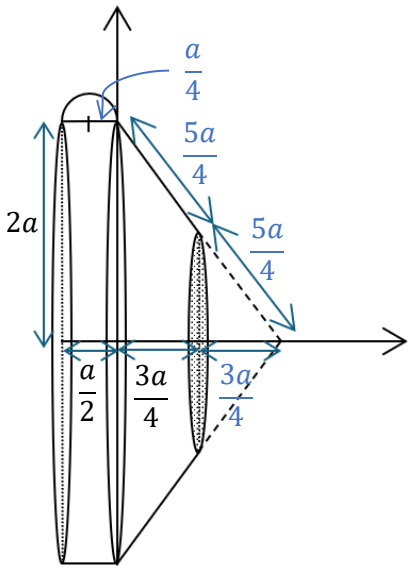
$$\bar{x} = \frac{\int_0^l x\cos\theta \cdot 2\pi\rho x \sin\theta\, dx}{\int_0^l 2\pi\rho x \sin\theta\, dx} \quad (5)\ (5)$$

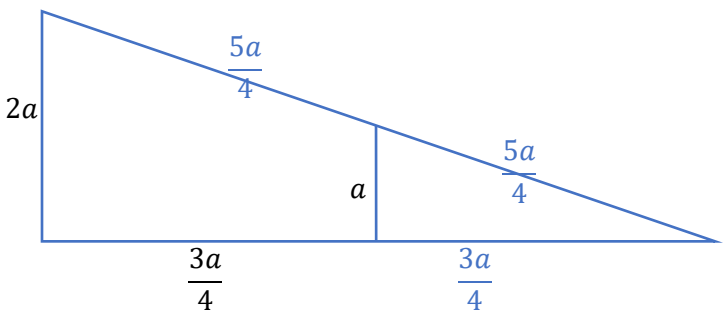
$$\bar{x} = \frac{\cos\theta \int_0^l x^2\, dx}{\int_0^l x\, dx}$$

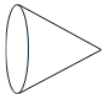
$$\bar{x} = \frac{\frac{h}{l}\left[\frac{x^3}{3}\right]_0^l}{\left[\frac{x^3}{3}\right]_0^l} \quad (5)$$

$$\bar{x} = \frac{2h}{3} \quad (5)$$

∴ Distance from the centre of the base to the centre of the mass is $\frac{h}{3}$ **50**





Object	Mass	Distance from y axis to C.O.G	Distance from x axis to
	$\pi a^2\rho$ (5)	$\frac{3a}{4}$ (5)	0
	$\pi \cdot 2a \cdot \frac{5a}{2} \cdot \rho = 5\pi a^2\rho$ (5)	$\frac{1}{3} \cdot \frac{3a}{2} = \frac{a}{2}$ (5)	0
	$\pi a \cdot \frac{5a}{4} \cdot \rho = \frac{5\pi a^2\rho}{4}$ (5)	$\frac{1}{3} \cdot \frac{3a}{4} + \frac{3a}{4} = a$ (5)	0
	$2\pi \cdot 2a \cdot \frac{a}{2} \cdot \rho = 2\pi a^2\rho$ (5)	$-\frac{1}{2} \cdot \frac{a}{2} = -\frac{a}{4}$ (5)	0
	$\pi \cdot \frac{a}{4} \cdot 4a\rho = \pi a^2\rho$ (5)	$-\frac{a}{4}$ (5)	$-\left(2a + \frac{2}{\pi} \cdot \frac{a}{4}\right)$ (5) $= -\left(2a + \frac{a}{2\pi}\right)$
Vessel	$\frac{31\pi a^2\rho}{4}$	$\overline{x}$	$\overline{y}$

$$\frac{31\pi a^2\rho}{4} \cdot \overline{y} = \pi a^2\rho \cdot \left(-\left(2a + \frac{a}{2\pi}\right)\right) \quad (10)$$

$$\overline{y} = -\frac{2a}{31\pi}(4\pi + 1) \quad (5)$$

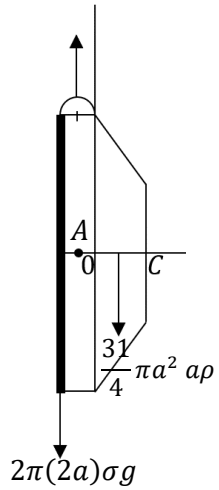
$$\frac{31\pi a^2\rho}{4} \cdot \overline{x} = \pi a^2\rho \cdot \frac{3a}{4} + 5\pi a^2\rho \cdot \frac{a}{2} - \frac{5\pi a^2\rho}{4} \cdot a + 2\pi a^2\rho \cdot \left(-\frac{a}{4}\right) + \pi a^2\rho \cdot \left(-\frac{a}{4}\right) \quad (10)$$

$$\frac{31}{4}\overline{x} = \frac{3a}{4} + \frac{5a}{2} - \frac{5a}{4} - \frac{2a}{4} - \frac{a}{4}$$

$$\overline{x} = \frac{5a}{31} \quad (5)$$

∴ Location of the Centre of the mass of the vessel $\equiv \left(\frac{5a}{31}, \frac{2a}{31\pi}(4\pi + 1)\right)$

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Let mass per unit length of the ring is σ

$$2\pi(2a)\sigma g \cdot \frac{a}{4} - \frac{31}{4}\pi a^2 \cdot \left(\frac{51a}{4 \cdot 31}\right) = 0 \quad (10)$$

$$\sigma = \frac{51}{16} \cdot a\rho \quad (5)$$

15

17. (a) A certain athlete participates in a track event consisting of several rounds and attempts to qualify for the final round. In the preliminary round, the probabilities that he obtains the first, second, and third positions are $\frac{1}{5}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. If the athlete obtains 1st 2nd or 3rd in the preliminary round, the probability that he qualifies for the final round is $\frac{3}{5}$ in each case. Otherwise, the probability that he qualifies for the final round is $\frac{1}{10}$. Find the probability that the athlete qualifies for the final round. **Given that,** the athlete has qualified for the final round, find the probability that he obtained one of the first three positions in the preliminary round.

(b) Let x_i and f_i be the class mark and the frequency of the i^{th} class interval of a grouped data distribution. It is **given** that the mean $\bar{x}$ and standard deviation σ of the distribution are $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$ and $\sigma = \sqrt{\frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}}$ respectively. If the observations are transformed using $y_i = ax_i + b$ where $a, b \in \mathbb{R}$, Show that the mean and the standard deviation of transformed data distribution are given by $\bar{y} = a\bar{x} + b$ and $\sigma_y = |a|\sigma$ respectively.

The Adjoining tables show summarized data collected by a researcher from the first 30 customers of a food court who preferred Food Z, on a particular day, with the aim of assisting the owner in making decisions to improve sales. Find the mean and the standard deviation of the customer's ages. **Hence,** deduce the mean expenditure on food Z and the corresponding standard deviation.

Ages of Customers (years)	Number of customers: Food Z
61–75	2
46–60	2
31–45	8
16–30	12
1–15	6

Expenditure (rupees)	Number of customers:
6100–7500	2
4600–6000	2
3100–4500	8
1600–3000	12
1100–1500	6

(a) Event of obtaining;

1st position - A $\quad P(A) = \frac{1}{5}$

2nd position - B $\quad P(B) = \frac{1}{4}$ (5)

3rd position - C $\quad P(C) = \frac{1}{3}$

(10) (5)

Any other position - D $\quad P(D) = 1 - \left(\frac{1}{5} + \frac{1}{4} + \frac{1}{3}\right) = \frac{13}{60}$

Qualifying for the finals - F

$P(F/A) = P(F/B) = P(F/C) = \frac{3}{5}$, $P(F/D) = \frac{1}{10}$ (5)

By total probability theorem, probability of qualifying for the finals $P(F)$,

$P(F) = P(F/A) \cdot P(A) + P(F/B) \cdot P(B) + P(F/C) \cdot P(C) + P(F/D) \cdot P(D)$ (10)

$P(F) = \frac{3}{5}\left(\frac{1}{5} + \frac{1}{4} + \frac{1}{3}\right) + \frac{1}{10} \cdot \frac{13}{60}$ (5)

$P(F) = \frac{59}{120}$ (5)

Probability that the athlete obtained one of the first three positions in the preliminary round and qualified for the final round $P(D'/F)$

$$P(D'/F) = \frac{P(F \cap D')}{P(F)} \text{ (5)} = \frac{P(F) - P(F \cap D)}{P(F)} \text{ (5)}$$

$$= 1 - \frac{P(F) - P(F/D) \cdot P(D)}{P(F)}$$

$$= 1 - \frac{\frac{1}{10} - \frac{1}{10} \cdot \frac{13}{60}}{\frac{59}{120}} \text{ (5)}$$

$$= \frac{282}{295} \text{ (5)}$$

65

$y_i = ax_i + b$

$f_i y_i = a(f_i x_i) + b f_i$ (5)

$\sum_{i=1}^{n} f_i y_i = a \sum_{i=1}^{n} (f_i x_i) + b \sum_{i=1}^{n} f_i$ (5)

$\frac{\sum_{i=1}^{n} f_i y_i}{\sum_{i=1}^{n} f_i} = a \frac{\sum_{i=1}^{n} (f_i x_i)}{\sum_{i=1}^{n} f_i} + b \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n} f_i}$ (5)

$\overline{y} = a\overline{x} + b$ (5)

$$\sigma_y{}^2 = \frac{\sum_{i=1}^{n} f_i(y_i - \overline{y})^2}{\sum_{i=1}^{n} f_i} \quad (5)$$

$$\sigma_y{}^2 = \frac{\sum_{i=1}^{n} f_i\big((ax_i + b) - (a\overline{x} + b)\big)^2}{\sum_{i=1}^{n} f_i} \quad (5)$$

$$\sigma_y{}^2 = \frac{a^2 \sum_{i=1}^{n} f_i\,(x_i - \overline{x})^2}{\sum_{i=1}^{n} f_i} \quad (5)$$

$$\sigma_y{}^2 = a^2\sigma^2 \quad (5)$$

$$\sigma_y = |a|\sigma$$

40

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(5) (5) (5)

Ages in (years)	Class mark z_i	f_i	$f_i z_i$	$z_i{}^2$	$f_i z_i{}^2$
61–75	68	2	136	4624	9248
46–60	53	2	106	2809	5618
31–45	38	8	304	1444	11552
16–30	23	12	276	529	6348
1–15	8	6	48	64	384
			$\sum_{i=1}^{5} f_i z_i = 870$		$\sum_{i=1}^{5} f_i z_i{}^2 = 33150$

Let age on food Z be α

$\therefore$ Mean age preferred food Z, $\overline{\alpha} = \frac{\sum_{i=1}^{5} f_i\, u_i}{\sum_{i=1}^{5} f_i\, u_i} = \frac{870}{30} = 29$ years (5)

Standard deviation on food Z, $\sigma_\alpha = \sqrt{\frac{\sum_{i=1}^{5} f_i {z_i}^2}{\sum_{i=1}^{5} f_i\,\overline{\alpha}} - \overline{\alpha}^2}$

$\sigma_\alpha = \sqrt{\frac{33150}{30} - 29^2}$ (5)

$\sigma_\alpha = \sqrt{1105 - 841}$

$\sigma_\alpha = \sqrt{264}$

$\sigma_\alpha = 16.25$ (5) **30**

Aliter

x_i	f_i	u_i	$f_i u_i$	$f_i {u_i}^2$
8	6	-2	-12	24
23	12	-1	-12	12
38	8	0	0	0
53	2	1	2	2
68	2	2	4	8
			-18	46

$A = 38, c = 15$

$u_i = \frac{x_i - 38}{15}$

$\overline{x} = A + c\left(\frac{\sum_{i=1}^{5} f_i\,u_i}{\sum_{i=1}^{5} f_i\,u_i}\right)$

$\overline{x} = 38 + 15\left(\frac{(-18)}{30}\right)$

$\overline{x} = 38 - 9$

$\overline{x} = 29$ years

$$\sigma^2 = c^2\left(\frac{\sum_{i=1}^{5} f_i {u_i}^2}{\sum_{i=1}^{5} f_i} - \left(\frac{\sum_{i=1}^{5} f_i u_i}{\sum_{i=1}^{5} f_i}\right)^2\right)$$

$$\sigma = 15\sqrt{\frac{46}{30} - \left(\frac{-18}{30}\right)^2}$$

$$\sigma = \frac{15}{30}\sqrt{46 \cdot 30 - 324} = 2\sqrt{66}\ years$$

Let the expenditure on food Z be y,

According to the distribution age of the customers and expenditure has a linear relation.

$y = 100\alpha$ (5)

In the transformation, $a = 100$ and $b = 0$

$\overline{y} = 100\overline{\alpha}$

$\overline{y} = 100 \cdot 29$ =Rs. 2900 (5)

$\sigma_y = |a|\sigma_\alpha$

$\sigma_y = 100 \cdot 16.25$ =Rs. 1625 (5)

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